Radiation is a mode of transfer of heat from one place to another without the requirement of the intervening medium which does not get heated during the process.

The radiant energy emitted by a body on account of its temperature is called thermal radiation. It is a particular range of electromagnetic radiation of wavelength ranging from 0.1 µm to 100 µm. This includes UV, visible and IR regions of EM spectrum.

Thermal radiation is an inherent property of all bodies. According to Prevost's theory of heat exchanges, every body absorbs and emits radiant energy continuously at all temperatures above absolute zero temperature (> 0 K).

Basic definitions

1 **Total Energy density** – The total radiant energy per unit volume around a point due to all wavelengths.

2 **Emissive power** of a body corresponding to a particular temperature and for a given wavelength is the amount of radiant energy emitted per unit time per unit surface area of the body within unit wavelength interval around λ. It is represented by e_λ.

3 **Emissivity** $(\in)$ of a body at a given temperature is the ratio of emissive power of the body (e) to the emissive power of perfectly black body (E) at that temperature.

4 **Absorptive power** of a body at a given temperature and for a given wavelength is the ratio of radiant energy absorbed per second per unit surface area of the body to the total energy falling per second on the same area. It is denoted by a_λ.

5 When thermal radiation is incident on a body (Q), a part of it is absorbed (Q_a), a part is reflected (Q_r) and the remaining part is transmitted (Q_t) through the body. If Q is the amount of incident radiation on a body, then $Q = Q_a + Q_r + Q_t$ …..(1)

Dividing (1) by Q, $1 = \frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q} = a + r + t$

Thus $1 = a + r + t$ where a, r and t are the absorption, reflection and transmission coefficients.

If t= 0, then a + r = 1, If a is large r is small. Good absorbers are bad reflectors. If r + t = 0, then a = 1, then the body is absorbing all incident radiation. Such a body is called a black body.

The values of r, a and t depends on nature of surface of the body and the wavelength of the incident radiation.

Perfect Black Body is that which absorbs all the radiations incident upon it. Thus absorptive power of a perfectly black body is unity (*i.e.* 100%). When such a body is heated to high temperature, it would emit radiations of all wavelengths called **black body radiation**.

The nature of radiations emitted by a perfectly black body would depend on its temperature only and not on mass, size, density or nature of the body. For an ideal black body, reflectance and transmittance must be zero. No body in actual practice can be perfectly black. The nearest examples of ideal black bodies are lamp black (96%) and platinum black (98%). They absorb visible and near infra red radiations, but cannot absorb far infra red radiations.

Kirchhoff's Law of radiation

According to this law, at a given temperature and for a given wavelength, the ratio of spectral emissive power $\left(e_\lambda \right)$ to spectral absorptive power $\left(a_\lambda \right)$ for all bodies is constant, which is equal to spectral emissive power of a perfectly black body $\left(E_\lambda \right)$ at the same temperature and for the same wavelength *i.e.*, $\frac{e_\lambda}{a_\lambda} = E_\lambda$ clearly, $e_\lambda \propto a_\lambda$ *i.e.* good emitters are good absorbers. The law implies that at a particular temperature, a body can absorb only those wavelengths, which it is capable of emitting. This law has been verified experimentally.

Let Q be the quantity of heat radiation incident on a body in one second. If Q_1 is the amount of radiation absorbed by the body and $e_\lambda \, d\lambda$ is the amount of heat energy radiated by the body in one second per unit area at a temperature T and wavelength λ, then the total energy given out by the body is $(Q - Q_1) + e_\lambda \, d\lambda$

$Q = (Q - Q_1) + e_\lambda \, d\lambda$ As $a_\lambda = \frac{Q_1}{Q}$ or $Q_1 = a_\lambda Q$

Thus $Q = (Q - a_\lambda Q) + e_\lambda \, d\lambda$ or $e_\lambda \, d\lambda = a_\lambda Q$

$\frac{e_\lambda}{a_\lambda} = \frac{Q}{d\lambda} = constant$. ……..(1)

For a perfect black body $e_\lambda = E_\lambda$ and $a_\lambda = 1$

Thus $\frac{E_\lambda}{1} = \frac{Q}{d\lambda}$ ……(2) Comparing (1) and (2) $\frac{e_\lambda}{a_\lambda} = E_\lambda = constant.$

Examples to illustrate Kirchhoff's law

1 When a green glass plate is heated in a furnace, it appears red. This is because, at ordinary temperatures green glass absorbs red strongly and emits green. At higher temperatures it emits red. Thus a good absorber is a good emitter.

2 A number of dark lines in the solar spectrum called Fraunhofer lines can be explained on the basis of Kirchhoff's law. When radiation from inner regions of sun pass through relatively cooler solar atmosphere having different gases, the gases absorb radiation of their characteristic wavelengths. These regions appear as dark lines. During total solar eclipse, these lines appear bright. By knowing the wavelengths of these lines, the presence of different elements in the solar atmosphere can be found.

Stefan's Law: According to this law, the total energy emitted per sec per unit area (E) by a perfectly black body corresponding to all wavelengths is directly proportional to fourth power of the absolute temp. (T) of the body. *i.e.*

$$E \propto T^4 \qquad \text{or} \qquad E = \sigma T^4$$

where σ is a constant of proportionality and is called *Stefan's constant.* Its value is

$$\sigma = 5.67 \times 10^{-8} \text{ watt } m^{-2}\ K^{-4}.$$

If Q is the total amount of heat energy emitted by the black body, then by definition,

$$E = \frac{Q}{A\,t} \qquad \therefore \qquad Q = A\,t \times E = A\,t\left(\sigma\,T^4\right)$$

Stefan Boltzmann Law According to this law, the net amount of radiation emitted per second per unit area of a perfectly black body at temp. T is equal to difference in the amounts of radiation emitted per sec per area by the body and by the black body enclosure at T_0. *i.e.* $E' = E - E_0$

As $E = \sigma T^4$ and $E_0 = \sigma T_0^4$

$\therefore$ $E' = \sigma T^4 - \sigma T_0^4 = \sigma\left(T^4 - T_0^4\right)$

Proceeding as above, total energy lost $Q' = E'\,A\,t = A\,t\,\sigma\left(T^4 - T_0^4\right)$

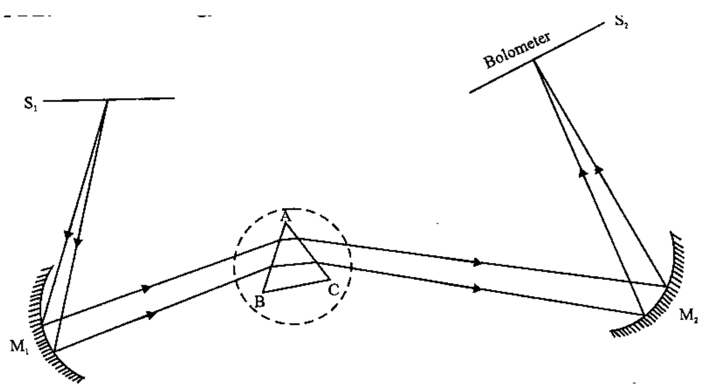
If the body and enclosure are not perfectly black and have emissivity *e*, then

$$Q' = e\ A\,t\,\sigma\left(T^4 - T_0^4\right)$$

Newton's Law of Cooling

According to this law, when difference in temperatures of a liquid and its surroundings is small, then the rate of loss of heat of the liquid is directly proportional to difference in temperature of liquid and the surrounding.

Black body radiation spectrum



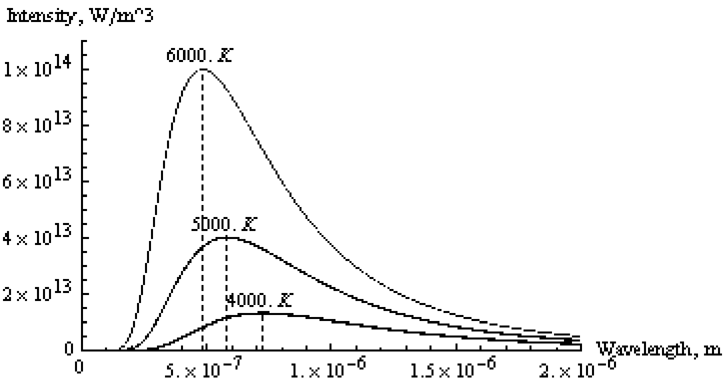
Lummer and Pringsheim investigated the distribution of energy emitted by a black body at different temperatures. The experimental set up used is as shown in the diagram.

The black body used is an electrically heated chamber with a small aperture. The temperature of the black body radiation emitted is measured using a thermocouple.

The radiation coming out of the slit S_1 is incident on the reflector M_1. The parallel rays from M_1 falls on a fluorspar prism placed on the turn table of the spectrometer. The emerging radiation is focused on a bolometer placed behind S_2 with the help of reflector M_2. The deflection produced by the galvanometer in the bolometer will determine the intensity of radiation. Using the prism dispersion formula, the wavelength of the radiation can be measured. Different wavelength radiations are deviated to different extents. The experiment is repeated for different temperatures.

A graph is plotted with wavelength of radiation along X axis and Intensity along Y axis as shown. It is referred to as black body radiation spectrum.

Experimental results



1 The black body radiation spectrum is not having uniform distribution of energy with respect to wavelength.

2 At a given temperature, different wavelength radiations have different energies. The magnitude of the emitted energy increases with increase in temperature.

3 The energy increases with wavelength, reaches a maximum value at a particular wavelength and decreases at higher wavelengths.

4 The total energy of radiation at any temperature is given by the area under the curve. The area is directly proportional to the fourth power of absolute temperature i.e. $E \propto T^4$. This is the Stefan's law.

5 The wavelength corresponding to maximum energy shifts towards shorter wavelengths with increase in temperature. This is the Wien's displacement law given by $\lambda_m \times T = constant$. Where λ_m is the wavelength corresponding to maximum energy.

The distribution of energy in the black body radiation spectrum was first explained by Wien with the help of two following laws.

<u>1 Wien's displacement Law:</u> According to this law, the wavelength (λ_m) corresponding to which energy emitted per sec per unit area by a perfect black body is maximum, is inversely proportional to the absolute temp. (T) of the black body.

$$\lambda_m \propto \frac{1}{T} \qquad \text{or} \qquad \lambda_m = \frac{b}{T}$$

where b is a constant of proportionality and is called Wien's constant $b = 2.898 \times 10^{-3}\ m\ K$.

Clearly,
$$\frac{\lambda_{m_1}}{\lambda_{m_2}} = \frac{T_2}{T_1} = \frac{\nu_{m_2}}{\nu_{m_1}}$$

<u>2 Wien's fifth power law :</u> According to this law, the maximum energy of emitted radiation E_m is directly proportional to the fifth power of absolute temperature i.e. $E_m \propto T^5$ or $E_m = K\,T^5$.

<u>Wien's distribution law</u> : With the help of above two laws, the expression for the energy density of radiation in the range λ and λ + dλ is given by the relation $E_\lambda\, d\lambda = C_1\, \lambda^{-5} f(\lambda, T)$

Or $$\boldsymbol{E_\lambda\, d\lambda = C_1\, \lambda^{-5}\, e^{-(C_2/\lambda T)}\, d\lambda} \qquad ....(1)$$

where C_1 and C_2 are constants. The above relation is called **Wien's distribution law of black body radiation spectrum**.

<u>Drawback of Wien's law</u> :The Wien's law is applicable only for the short wavelength region and for high temperature of the source of radiation. It fails to explain the decrease in the energy for longer wavelengths.

Raleigh Jeans law

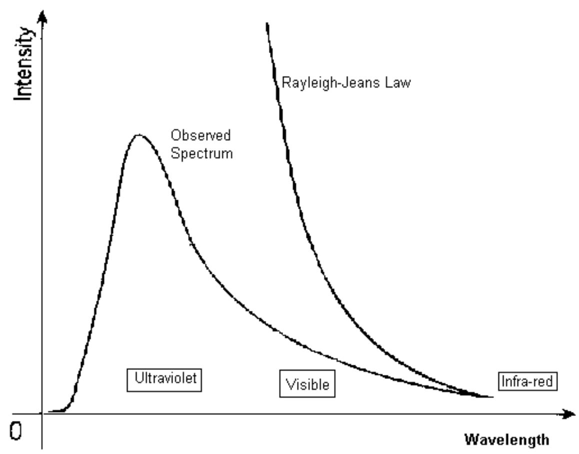
According to Raleigh and Jeans, the radiation emitted by a particle in the black body travels as waves. There are several such waves which undergo reflections from the walls of the enclosure. These waves superpose and produce stationary waves. The frequencies of vibrations of the system called the modes of vibration per unit volume is given by = $\frac{8\pi\, d\lambda}{\lambda^4}$.

It is assumed that the law of equipartition of energy is valid here. According to this law average energy per mode of vibration is kT, where k is the Boltzmann constant and T is the absolute temperature.

Thus the energy density within the range λ and λ + dλ is given by

$\boldsymbol{E_\lambda\, d\lambda = \frac{8\pi\, d\lambda}{\lambda^4}\, kT = \frac{8\pi\, kT}{\lambda^4}\, d\lambda}$. This is **Raleigh Jeans law for energy distribution.**

As $\nu = \frac{c}{\lambda}$ implies $\lambda = \frac{c}{\nu}$ differentiating, dλ = $\left|\frac{c}{\nu^2}\right| d\nu$. Thus energy density in terms of frequency is $E_\nu\, d\nu$ = $\frac{8\pi\nu^2 kT}{c^3}\, d\nu$



<u>Drawback of Raleigh Jeans law</u> :The energy density is given by

$\boldsymbol{E_\lambda\, d\lambda = \frac{8\pi\, kT}{\lambda^4}\, d\lambda}$. For shorter wavelengths, the energy increases. As the wavelength tends to zero, the energy increases continuously and goes to infinity. This is called **<u>ultraviolet catastrophe</u>**. This theory agrees for longer wavelengths. At shorter wavelengths there is disagreement between the experiment and theory.

Planck's law of radiation

The failure of both Wien's law and Raleigh Jeans law led Max planck to develop a theory to explain the black body radiation spectrum. He made the following assumptions.

1 The black body is made up of a large number of oscillating particles (simple harmonic oscillators). These particles can vibrate in all possible frequencies.

2 The oscillators can have only discrete set of energies which is the integral multiple of a finite quantum of energy.

3 The energy is given by $\varepsilon = n\,h\nu$ where ν is the frequency of oscillator and h is the planck's constant. n is a integer called quantum number. Thus the energy of an oscillator can be only $h\nu$, $2h\nu$, $3h\nu$,…….. Thus energy of the oscillator is quantised.

4 An oscillator emits or absorbs energy discontinuously and not continuously as given by electromagnetic theory. The absorption or emission occurs as quantum of energy $h\nu$.

Expression for Average energy of planck's oscillator – Planck's law of radiation

Ler N_0, N_1, N_2,………N_n be the number of oscillators having energies 0, $h\nu$, $2h\nu$, $3h\nu$,……..n $h\nu$ respectively in a body.

The total number of oscillators is $N = N_0 + N_1 + N_2 + \ldots\ldots\ldots N_n$ ….(1)

The total energy of the oscillators is

$$E = N_0(0) + N_1\, h\nu + N_2\, 2\,h\nu, + N_3\, 3\,h\nu + \ldots\ldots. N_n\, n\, h\nu.$$

If $\varepsilon = h\nu$, then $E = 0 + \varepsilon + 2\,\varepsilon + 3\,\varepsilon + \ldots\ldots. n\,\varepsilon$ …..(2)

The relative probability that an oscillator has energy $h\nu$ at temperature T is given by Boltzmann factor $e^{-\,h\nu/kT}$. Thus according to Boltzmann distribution law, the number of oscillators having energy $h\nu$ is

$$N_1 = N_0\, e^{-\,h\nu/kT}$$

Similarly number of oscillators having energy $nh\nu$ is $N_n = N_0 e^{-\,nh\nu/kT}$.

Thus equation (1) becomes

$$N = N_0\, e^{-0\,h\nu/kT} + N_0\, e^{-\,h\nu/kT} + N_0\, e^{-\,2h\nu/kT} + \cdots \ldots + N_0\, e^{-\,nh\nu/kT}$$

$$N = N_0\left(1 + e^{-\,h\nu/kT} + e^{-\,2h\nu/kT} + \cdots \ldots + e^{-\,nh\nu/kT}\right)$$

Let $e^{-\,h\nu/kT} = y$

Then $N = N_0(1 + y + y^2 + \cdots \ldots + y^n)$

As $1 + y + y^2 + \cdots \ldots + y^n = \frac{1}{(1-y)}$

Thus $N = N_0 \times \frac{1}{(1-y)}$ ….. (3)

The total energy of the oscillators is

$$E = N_0(0) + N_1\, h\nu + N_2\, 2\,h\nu, + N_3\, 3\,h\nu + \ldots\ldots. N_n\, n\, h\nu.$$

Or E = N_1 ε +N_2 2 ε +N_3 3 ε + …….N_n n ε where $\varepsilon = h\nu$.

Or E = $N_0\, e^{-h\nu/kT}\varepsilon + N_0\, e^{-2h\nu/kT}2\varepsilon + \cdots \ldots + N_0\, e^{-nh\nu/kT}$nε

E = $N_0\varepsilon\left(e^{-h\nu/kT} + e^{-2h\nu/kT}2 + \cdots \ldots + e^{-nh\nu/kT}\text{n}\right)$

As $e^{-h\nu/kT} = y,$ Thus E = $N_0\varepsilon\ (y + 2\,y^2 + \cdots \ldots + \text{ny}^n\ )$

Or E = $N_0\varepsilon y\ (1 + 2\,y + 3y^2 \ldots \ldots + \text{ny}^n \ldots . .)$

As $1 + 2\,y + 3y^2 \ldots \ldots = \frac{1}{(1-y)^2},$

Thus E = $N_0\varepsilon y \times \frac{1}{(1-y)^2}$ ……….. (4)

The average energy of the oscillators is $\bar{E} = \frac{E}{N}$ ……(5)

substituting for E and N from (4) and (3) in (5) we get

$$\bar{E} = \frac{E}{N} = N_0\varepsilon y \times \frac{1}{(1-y)^2} \times \frac{(1-y)}{N_0} = \frac{\varepsilon y}{(1-y)}$$

Thus $\bar{E} = \frac{h\nu\, e^{-h\nu/kT}}{\left(1 - e^{-h\nu/kT}\right)} = \frac{h\nu}{e^{h\nu/kT} - 1}$

The average energy of an oscillator is $\boldsymbol{\bar{E} = \frac{h\nu}{e^{(h\nu/kT)}-1}}$ and not kT as predicted by the classical theory.

The number of modes of vibration in the frequency range ν and ν + dν is equal to $\frac{8\,\pi\nu^2}{c^3}\ d\nu$. Multiplying this relation with the expression for average energy, we get the energy density.

Thus $\boldsymbol{E_\nu\, d\nu = \frac{8\,\pi\nu^2}{c^3}\frac{h\nu}{e^{(h\nu/kT)}-1}\ d\nu}$. This is called the **planck's law of radiation**. This law can be expressed in terms of wavelength as follows,

As ν $= \frac{c}{\lambda},$ differentiating, dν $= \left|\frac{c}{\lambda^2}\right| d\lambda$

Thus the energy density of radiation in the wavelength range λ and λ + dλ is given by $E_\lambda\, d\lambda = \frac{8\,\pi h}{c^3}\left(\frac{c^3}{\lambda^3}\right) \times \frac{1}{\left(e^{(hc/\lambda kT)}-1\right)}\left(\frac{c}{\lambda^2}\right)\ d\lambda$

Or $$\boldsymbol{E_\lambda\, d\lambda = \frac{8\,\pi hc}{\lambda^5}\frac{1}{\left(e^{(hc/\lambda kT)}-1\right)}d\lambda}$$

This formula agrees well with the experimental observation of black body radiation spectrum.

To derive Wien's formula from Planck's law of radiation

When λ is very small $e^{(hc/\lambda kT)}$ is very large compared to 1 .

In the planck's formula $E_\lambda \, d\lambda = \frac{8\,\pi hc}{\lambda^5}\frac{1}{\left(e^{(hc/\lambda kT)}-1\right)} d\lambda,$ 1 can be neglected compared to $e^{(hc/\lambda kT)}$.

Thus $E_\lambda \, d\lambda = \frac{8\,\pi hc}{\lambda^5}\frac{1}{\left(e^{(hc/\lambda kT)}\right)} d\lambda$ or $E_\lambda \, d\lambda = \frac{8\,\pi hc}{\lambda^5} e^{-(\mathrm{hc/\lambda kT})} d\lambda$

Let $8\pi hc = C_1$ and $\frac{hc}{k} = C_2$, Then the above equation can be written as

$\boldsymbol{E_\lambda \, d\lambda = \frac{C_1}{\lambda^5} e^{-(C_2/\lambda \mathrm{T})}} d\lambda$ This is the Wien's law which agrees with the experimental results at shorter wavelengths.

To derive Raleigh Jeans law from Planck's law of radiation

For longer wavelengths, i.e. when λ is very large, $hc/\lambda kT$ is small.

Expanding $e^{\left(\frac{hc}{\lambda kT}\right)}$, we get $e^{\left(\frac{hc}{\lambda kT}\right)} = 1 + \frac{hc}{\lambda kT} + \left(\frac{hc}{\lambda kT}\right)^2 + \ldots \ldots$

Neglecting the higher powers, we get $e^{\left(\frac{hc}{\lambda kT}\right)} = 1 + \frac{hc}{\lambda kT}$

From the planck's law $E_\lambda \, d\lambda = \frac{8\,\pi hc}{\lambda^5}\frac{1}{\left(e^{(hc/\lambda kT)}-1\right)} d\lambda$. Putting tha above condition in this equation,

$E_\lambda \, d\lambda = \frac{8\,\pi hc}{\lambda^5}\frac{1}{\left(1+\frac{hc}{\lambda kT}-1\right)} d\lambda = \frac{8\,\pi hc}{\lambda^5}\frac{1}{\frac{hc}{\lambda kT}} d\lambda$

Thus $E_\lambda \, d\lambda = \frac{8\,\pi hc}{\lambda^5}\frac{\lambda kT}{hc} d\lambda$ Or $\boldsymbol{E_\lambda \, d\lambda = \frac{8\,\pi kT}{\lambda^4} d\lambda}$

This is the Raleigh Jeans law which agrees well with experimental observation for longer wavelengths.

Radiation pressure

Electromagnetic waves transport momentum as well as energy. When this momentum is absorbed by a surface, pressure is exerted on it. This is referred to as **radiation pressure.**

Let a photon of energy hν moving with the velocity of light c, incident on a surface of a body along the normal. From Einstein theory of relativity, $E = mc^2$ the mass of the photon is $m = E/c^2$ *or* $m = h\nu/c^2$.

Thus the momentum of the photon is $p = m \times c$ or $p = h\nu/c^2 \times c = h\nu/c$.

or $= \frac{\varepsilon}{c}$.

The total momentum incident on the surface along the normal is given by

$$p = \frac{\sum \varepsilon}{c} = \frac{E}{c}\ .$$

If u is the energy density i.e. energy per unit volume, then the total energy incident on a surface of area s in unit time along the normal is given by $E_1 = u\ s\ c$.

Thus the energy flux, i.e. radiation per unit area is $E = E_1/s$.

Thus $E = \frac{u\,s\,c}{s} = u\ c$.

The rate of momentum transfer per unit area $p = \frac{E}{c} = \frac{u\,c}{c} = u$.

As rate of momentum transfer is force and force per unit area is pressure, thus the radiation pressure $\boldsymbol{P = u}$.

For normal incidence, the radiation pressure is equal to the energy density.

For diffused radiation, pressure is given by $P = u/3$.

Solar constant

Sun emits radiation continuously in all directions. A large portion of energy is lost due to scattering, absorption and reflection during propagation through earth's atmosphere.

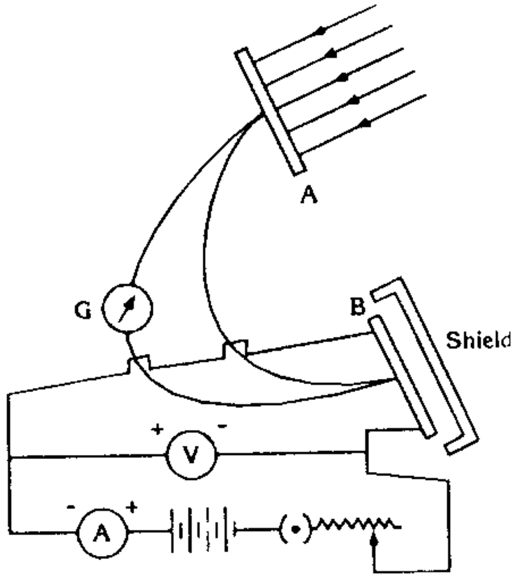
The rate at which solar radiation is received by unit area of a black body placed at right angles to the incoming radiation at the mean distance of the earth from the sun in the absence of earth's atmosphere is called solar constant.

Its value is $1.35 kWm^{-2}$. It is experimentally determined by a device called pyrheliometer.

Angstrom's Pyrheliometer

Principle : The solar energy absorbed by one metal strip is balanced or compensated by a known electrical energy supplied to another identical one.

Apparatus and working: It consists of two identical thin metal strips made of manganin or constantan A and B which are blackened. A is exposed to sun's radiation and B is shielded. A copper-constantan thermocouple



is attached with its junctions to the mid-points of A and B, with a sensitive galvanometer G. The junctions are electrically insulated and placed so close to A and B such that they attain the same temperature as A and B.

When A is exposed to solar radiation, its temperature rises and the galvanometer will show a deflection. Now, the temperature of B is increased by passing a current through it. The current is so adjusted that the galvanometer deflection becomes zero. This means that the rate at which A absorbs the heat energy is equal to the rate at which heat is supplied to B.

If I is the current and V is the potential difference then the electrical energy supplied is VI joules in one second.

If A is the area of the exposed strip, a its absorption coefficient, S, the incident radiation per unit area per second, the radiant energy absorbed is $S\,a\,A = V\,I$, *or the solar constant* $\boldsymbol{S} = \frac{\boldsymbol{V\,I}}{\boldsymbol{a\,A}}$ Wm^{-2}.

Surface temperature of Sun

Consider the sun to be a perfect black body of radius R and surface temperature T, then the amount of heat radiated by the sun per second is given by

$$Q = 4\pi R^2\,\sigma\,T^4 \quad \ldots\ldots(1)$$

Where σ is the Stefan's constant. This energy spreads over a surface area $4\pi\,r^2$ where r is the mean distance of the earth from the sun. Hence the solar constant

$$S = \frac{Total\ heat\ radiated\ per\ second}{area} \qquad \text{Thus } S = \frac{4\pi R^2\,\sigma\,T^4}{4\pi r^2} \qquad \text{or} \qquad T^4 = \frac{S}{\sigma}\left(\frac{r}{R}\right)^2$$

The surface temperature of the sun is given by $\boldsymbol{T} = \left[\frac{\boldsymbol{S}}{\boldsymbol{\sigma}}\left(\frac{\boldsymbol{r}}{\boldsymbol{R}}\right)^{\boldsymbol{2}}\right]^{\boldsymbol{1/4}}$

Substituting the following data in the above equation, T can be calculated.

$R = 6.928 \times 10^8$ m, $r = 1.5 \times 10^{11}$ m, $\sigma = 5.7 \times 10^{-8}\ Wm^{-2}K^{-4}$ and $S = 1.388 \times 10^3\ Wm^{-2}$ The value of T is found as **T = 5780 K**. This temperature is called black body temperature as sun is assumed to be black body or effective temperature of sun.