II B. Sc. Mathematics

III Semister, MODEL PAPER - I

I. Answer any FIFTEEN question :- (2 x15 = 30)

1. Find the order of each elements of the Group{1, -1, i, -i} w.r.t. multiplication
2. Show that every cyclic group is abelian.
3. Show that the multiplicative group G = $\{1, w, w^2\}$ is cyclic
4. Find the number of generators of cyclic group of order 60
5. Write all the left cosets of H = {0, 4, 8} the subgroup of the additive group of integers modulo 12.
6. Prove that a finite group of prime order does not have proper sub groups.
7. Define convergent and divergent of a sequence.
8. 'Every convergent sequence is bounded' Give an example.
9. Verify cauchy's criterion for the sequence $\left\{\frac{2n-3}{4n+5}\right\}$
10. Discuss the convergence of $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+------\infty$
11. State D' Alembert's Ratio test.
12. Prove that $\sum\left(\frac{n+1}{n}\right)^{2n^2}$ is divergent.
13. Discuss the convergence of x$-\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+--\infty$, x<1.
14. Find the summation of $\sum\frac{1}{(2n)!}$
15. Prove that $\underset{x\to 0}{Lt}\left(\frac{x^2+5x+3}{x^2+x+2}\right)^x = e^4$

16. Give an example for a function which is continuous but not differentiable
17. Show that Sinx. log Sinx satisfies all the conditions of Rolle's Theorem. in $[0, \frac{p}{2}]$
18. Evaluate $\underset{x\to 0}{Lt}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}$
19. Calculate a_0 in the Fourier Series expansion of e^{-x} in (-1, 1).
20. If f(x) = $\begin{cases} 0 & for\ \ -2<x<0 \\ 1 & for\ \ 0<x<2 \end{cases}$

Find the Fourier coefficient a_n in the Fourier Series.

II Answer any three of the following :- (3 x 5 = 15)

1. Define order of a group. Show that the order of any power of an element of a group cannot exceed the order of the element.
2. In a group , if ba = $a^m b^m$ prove that the same order.
3. Prove that in a cyclic group G = $\langle a \rangle$ of order K, $a^m = a^n$ (m≠n) then $m \equiv n$ (mod K) and conversely.
4. Prove that any two right cosets of a sub group H of a group are either disjoint or identical.
5. In a group G, $a^5 = e$ and $b^2 = aba^{-1}$ prove that $b^{31} = e$

III Answer any two of the following:- (2 x 5 = 10)

1. Prove that { $n^{1/n}$} is convergent.
2. Show that the sequence {x_n} defined by $x_1 = \sqrt{5}$ and $x_{n+1} = \sqrt{5+x_n}$ converges to the +ve root of the equation $x^2 - x - 5 = 0$.
3. When $\underset{n\to\infty}{Lt}\frac{3n-4}{5n+7}=\frac{3}{5}$, $\in = \frac{1}{10}$ find the corresponding N.

IV Answer any three of the following : (3 x 5 = 15)

1. Discuss the convergence of the series $\sum_{1}^{\infty} \frac{1}{n^{p}}$
2. Show that $\frac{5}{1.2.3}+\frac{7}{3.4.5}+\frac{9}{5.6.7}+---\infty$ is convergent.
3. Discuss the convergence of the series

   $\frac{x}{2\sqrt{3}}+\frac{x^{2}}{3\sqrt{4}}+\frac{x^{3}}{4\sqrt{5}}+---\infty$
4. Find sum of $\frac{1}{1.2.3}+\frac{1}{3.4.5}+\frac{1}{5.6.7}+---\infty$
5. If S = $\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+--\infty$ prove that $16S^2 - 56S - 79 = 0$

V Answer any two of the following (2 x 5 = 10)

1. Discuss the type of discontinuity of

   $$f(x) = \begin{cases} \frac{x^2-4x+3}{x^2-5x+4} & x \neq 1 \\ 3/2 & x = 1 \end{cases}$$
2. State and prove Rolle's Theorem.
3. Verify Lagranges Mean value theorem for

   F(x) Sinx +cotx in $[0, \frac{p}{2}]$
4. Show that log sin(x+h) = log sinx + hcotx - $\frac{h^2}{2}$ cosec^2x +--

VI Answer any Two of the following :- (2 x 5 = 10)

1. Find the Fourier series for f(x) = $|x|$ in (-1 , l)
2. find the half – range Sinx series for f(x) = p x – x^2 in 0<x<p
3. Find the half – range cosine series for f(x) $(x-1)^2$ in 0<x<1

MODEL PAPER II

I Answer any FIFTEEN questions :- (2 x 15 = 30)

1. If 0(a) = m, 0(b) = n ∀ a, b∈ G an abelian group, (m,n) = 1 prove that 0(ab) = mn.
1. Define a cyclic group.
2. If a is the generator of a cyclic group then show that a^{-1} is also a generator.
3. Show that G = {2^n | n∈ z} w.r.t usual multiplication is a Cyclic group.
4. Define coset of a group
5. Find the idex of the sub-group H = {0,3} in the group $\langle Z_6 + \rangle$
6. Show that a monotonic decreasing sequence bounded below is convergent.
7. Give an exa mple for the statement sum of the two divergent sequences is divergent
8. Prove that the sequences {x_n} whose n^{th} term is

   $x_n = \frac{2n^2+3n+5}{n+3}.\sin\frac{p}{n}$ is convergent.
9. Discuss the convergence of $\sum (\log n)^{-n}$
10. State Raabe's test for a positive series.
11. Discuss the convergence of $\sum \frac{2.4.6.......(2n)}{3.5.7........(2n+1)}$
12. Show that 1- $\frac{1}{\sqrt[3]{2}}$ + $\frac{1}{\sqrt[3]{3}}$ - $\frac{1}{\sqrt[3]{4}}$ + - - - is convergent.
13. Find the sum of 1 + $\frac{3}{1!}\frac{1}{3}+\frac{3.5}{2!}(\frac{1}{3})^2$ + - - - - ∞
14. If a and b are the roots of x^2 – ax +b = 0 prove that

$$\underset{x\to a}{Lt}\frac{e^{(x^2-ax+b)}-1}{x-a}=a-b$$

15. Show that f(x) = $e^{-|x|}$ is not differentiable at x = 0.
16. Verify Lagranges Mean value theorem for
    f(x) = (x-1) (x-2) in [0, 4].
18. State Taylors Expansion for a differentiable function
19. Obtain the half – range sine series for f(x) = x over the interval (0, π).
20. Obtain the Fourier Series for the function f(x) = x^2 over the interval (- π, π).

II Answer any three of the following :- (3 x 5 = 15)

1. Define center of a group. Prove that center of a group Is a sub-group of G.
2. Show that $\left\{I_2\begin{bmatrix}-1 & 1\\ -1 & 0\end{bmatrix},\begin{bmatrix}0-1\\ 1-1\end{bmatrix}\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}1-1\\ 0-0\end{bmatrix}\begin{bmatrix}0 & 1\\ -1 & 1\end{bmatrix}\right\}$
   It forms a cyclic group.
3. Prove that there is a one – to one correspondence between the set of all right cosets and the set of all left cosets of a sub group of a group.
4. State and prove Lagranges theorem for a finite group.
5. Find all the district cosets of the sub group H = $\{1,3,9\}$ of the group G = $\{1,2,3,---12\}$ w.r.t x mod 13.

III Aanswer any two of the following :- (2 x 5 = 10)

1. If $\{a_n\}\to a$ and $\{b_n\}\to b$, prove that $\{a_n+b_n\}\to a+b$.
2. Show that the sequence $\{x^n\}$ where $x_n=\frac{3n+4}{2n+1}$ is
   (i) monotonic decreasing (ii) bounded (iii) tends to the lt $\frac{3}{2}$
3. Discuss the convergence of the following sequence
   (i) $\left\{\sqrt{n}(\sqrt{n+4}-\sqrt{n})\right\}$ (ii) $\{1+\cos np\}$

IV Answer any three of the following : (3 x 5 = 15)

1. State and prove D' Alembert's test of convergenc
2. Discuss and convergence of
   a) $\sum\frac{1}{n}\tan\frac{1}{n}$ b) $\frac{1}{1^p}+\frac{1}{3^p}+\frac{1}{5^p}+---$
3. Discuss the convergence of $x+\frac{x^2}{\sqrt{2}}+\frac{x^3}{\sqrt{4}}-+---\infty$
4. Sum the series $\frac{3^2}{1!}+\frac{5^2}{3!}\frac{7^2}{5!}+---$
5. Prove that $1+\frac{1}{4}\left(\frac{1}{2}+\frac{1}{3}\right)+\frac{1}{4^2}\left(\frac{1}{4}+\frac{1}{5}\right)--\infty=\frac{1}{2}\log 12$

V Answer any two of the following : - (2 x 5 = 10)

1. A function which is continuous in a closed interval attain its bounds
2. Verify Cauchy's Mean Value theorem for
   f(x) = $\sqrt{x}$ and $g(x)=\frac{1}{\sqrt{x}}$ in [a, b]

1. Find the Maclaurins expansion of e^x cosx.
2. Find the values of a,b,c so that $\underset{x\to 0}{Lim}\frac{axe^x-b\log(1+x)+cxe^{-x}}{x^2\sin x}=2$

VI Answer any two of the following : - (2 x 5 = 10)

1. Find the fourier for f(x) = x sin x, 0 < x < 2*p*
2. Find the half range cosine series for

$$f(x) = \begin{cases} x & 0 < x < \dfrac{a}{2} \\ a - x & \dfrac{a}{2} < x < a \end{cases}$$

3. Find the half range sine series for

   f(x) = 2x – 1 in the interval (0, 1)

V. Answer any two of the following:- (2 x 5 = 10)

1. A function which is continuous in a closed interval attain its bounds.
2. Verify Cauchy's Mean Value theorem for

   $f(x) = \sqrt{x}$ and $g(x) = \dfrac{1}{\sqrt{x}}$ in [a, b].
3. Find the Maclaurins expansion of $e^x \cos x$.
4. Find the values of a, b, c so that

$$\lim_{x \to 0} \frac{axe^x - b\log(1+x) + cxe^{-x}}{x^2 \sin x} = 2$$

VI Answser any two of the following :- (2 x 5 = 10)

1. Find the fourier series for f(x) = xsinx, $0 < x < 2\pi$.
2. Find the half range cosine series for

$$f(x) = \begin{cases} x & 0 < x < \dfrac{a}{2} \\ a - x & \dfrac{a}{2} < x < a. \end{cases}$$

3. Find the half range sine series for

   f(x) = 2x – 1 in the interval (0, 1).