Syllabus : Unit 1- Newton's laws and their applications

- Statement and explanation of the Newton's laws of motion, Inertial frames of reference, Galilean transformations, Atwood machine, Static and dynamic friction, Motion along inclined plane with and without frictional force, Use of free body diagrams, motion in a resistive medium, terminal velocity.
- Non-inertial frames of reference, Rotating coordinate system, pseudo forces, Centrifugal and Coriolis forces, effects of Centrifugal and Coriolis forces ai earth's surface, the Foucault pendulum (qualitative)

Newton's Laws of Motion : Newton's First Law :

Statement : "A body remains in a state of rest or of uniform motion unless acted upon by an external force (or unbalanced force)."

Explanation and significance :

- Newton's first law is often referred to as the **law of inertia**.
- Inertia is defined as the tendency of an object to remain at rest or move with a constant velocity Inertia is a fundamental property of all matter and is important to the definition of mass.
- Mass is that quantity which is the measure of inertia. The more inertia that an object has, the more mass that it has.
- To change the state of rest or state of uniform motion of a body, an external force is necessary.

Consider the following example. A string with its one end connected to a ball and the other end held in hand is made to execute a circular motion. The string must provide the necessary centripetal force to move the ball in a circle. If the string breaks, the ball will move off in a straight line. The straight line motion in the absence of force is an example of Newton's first law. This implies that a force is necessary to change the state of rest or of uniform motion of a body.

Examples : 1. If you kicked a ball in space, it would keep going forever, because there is no gravity, friction or air resistance going against it. It will only stop going in one direction if it hits something like a meteorite or reaches the gravity field of another planet. 2. If you are driving in your car at a very high speed and hit something, like a brick wall or a tree, the car will come to an instant stop, but you will keep moving forward. This is why cars have airbags, to protect you from smashing into the windscreen

Newton's Second law :

Statement : "The net force on an object is equal to the mass of the object multiplied by its acceleration." (**F** = m**a**). Also, the net force on a particle is equal to the time rate of change of its linear momentum:

Explanation : Mathematically, it is given by $F = \frac{dp}{dt}$, where $p = mv$(constant of proportionality is taken as one). Thus $F = \frac{d}{dt}(mv)$. Assuming m to be a constant, the above equation is$F = m\frac{dv}{dt}$,As $a = \frac{dv}{dt}$, Thus $\boldsymbol{F = ma}$

If $F = 0$, then from above equation, $0 = m\frac{dv}{dt}$ or $\frac{dv}{dt} = 0$or $v = constant$. This is Newton's first law.

Significance : Newton's Second Law says that the net force, **F**, acting on an object causes the object to accelerate. Since **F** = m**a** can be rewritten as **a** = **F**/m, you can see that the magnitude of the acceleration is directly proportional to the net force and inversely proportional to the mass, m. Both force and acceleration are vector quantities, and the acceleration of an object will always be in the same direction as the net force.

Examples : 1. If you use the same force to push a truck and push a car, the car will have more acceleration than the truck, because the car has less mass. 2. It is easier to push an empty shopping cart than a full one, because the full shopping cart has more mass than the empty one. This means that more force is required to push the full shopping cart.

Newton's Third law:

"To every action there is an equal and opposite reaction."

Explanation : Whenever a particle A exerts a force on another particle B, B simultaneously exerts a force on A with the same magnitude but in the opposite direction. The law further postulates that these two forces act along the same line.

Significance : The Third Law means that all forces are interactions, and there is no such thing as a unidirectional force. If body A exerts a force on body B, simultaneously, body B exerts a force of the same magnitude on body A, both forces acting along the same line. It is important to note that the action and reaction act on different objects and do not cancel each other.

The two forces in Newton's third law are of the same type (e.g., if the road exerts a forward frictional force on an accelerating car's tires, then it is also a frictional force that Newton's third law predicts for the tires pushing backward on the road.

Examples : 1. When you jump off a small rowing boat into water, you will push yourself forward towards the water. The same force you used to push forward will make the boat move backwards. 2. When air rushes out of a balloon, the opposite reaction is that the balloon flies up. 3. When you dive off of a diving board, you push down on the springboard. The board springs back and forces you into the air.

Forces are of two types :1. Contact forces are those types of forces that result when the two interacting objects are perceived to be physically in contact with each other. Examples of contact forces include frictional forces, tensional forces, normal forces, air resistance forces, and applied forces.

2. Action-at-a-distance forces are those types of forces that result even when the two interacting objects are not in physical contact with each other, Examples of action-at-a-distance forces include gravitational forces, electrical and magnetic forces.

Types of forces

1. **The force of gravity** is the force with which the earth, moon, or other massively large object attracts another object towards itself. By definition, this is the weight of the object. All objects upon earth experience a force of gravity that is directed "downward" towards the center of the earth. The force of gravity on earth is always equal to the weight of the object given by $w = mg$.
2. **The normal force** is the support force exerted upon an object that is in contact with another stable object. For example, if a book is resting upon a surface, then the surface is exerting an upward force upon the book in order to support the weight of the book.
3. **The friction force** is the force exerted by a surface as an object moves across it or makes an effort to move across it. The friction force often opposes the motion of an object. For example, if a book slides across the surface of a desk, then the desk exerts a friction force in the opposite direction of its motion. Friction results from the two surfaces being pressed together closely, causing intermolecular attractive forces between molecules of different surfaces.
4. **The air resistance** is a special type of frictional force that acts upon objects as they travel through the air. The force of air resistance is often observed to oppose the motion of an object. This force will frequently be neglected due to its negligible magnitude It is most noticeable for objects that travel at high speeds (e.g., a skydiver or a downhill skier) or for objects with large surface areas.
5. **The tension force** is the force that is transmitted through a string, rope, cable or wire when it is pulled tight by forces acting from opposite ends. The tension force is directed along the length of the wire and pulls equally on the objects on the opposite ends of the wire.
6. **The spring force** is the force exerted by a compressed or stretched spring upon any object that is attached to it. An object that compresses or stretches a spring is always acted upon by a force that restores the object to its rest or equilibrium position.

Note :

- Mass is the quantity of matter in an object. More specifically, mass is a measure of the inertia, or "laziness," that an object exhibits in response to any effort made to start it, stop it, or otherwise change its state of motion.
- Weight is the force of gravity on an object.
- If force is equal to mass x acceleration then, Weight is equal to mass × acceleration due to gravity. On earth, your weight is Your Mass × 9.8 ms^{-2}

Frames of Reference

Rest and Motion

- A body is said to be at rest if its position does not change with time with respect to an observer (or a reference point).
- A body is said to be in motion if its position changes with time with respect to an observer (or a reference point).
- Rest and motion are relative terms. A body may seem to be at rest with respect to one object, but may appear to be in motion with respect to another object.
- If you consider a passenger in a moving train, he is at rest with respect to his co-passengers, but is in motion with respect to an observer standing on the ground.

A system of co-ordinate axis which defines the position of a particle in two or three dimensional space is called a **frame of reference**. The simplest frame of reference is the Cartesian system of co-ordinates, in which the position of the particle is specified by its three co-ordinates *x, y, z* along the three perpendicular axes.

In general, it is a framework that is used for the observation and mathematical description of physical phenomena and the formulation of physical laws. It consists of an observer, a coordinate system, and clocks assigning times at positions with respect to the coordinate system.

Inertial frame of reference	Non-inertial frame of reference
The reference frames in which Newton's laws are valid.	Reference frames in which Newton's laws are not valid.
They are non-accelerating frames	They are accelerating frames.
Such a constant velocity frame of reference is called an inertial frame because the law of Inertia holds in it.	Such an accelerating frame of reference is called a non-inertial frame because the law of inertia does not hold in it.
In an inertial frame of reference no fictitious forces arise.	In a non-inertial frame of reference fictitious forces arise.

Here zero force corresponds to zero acceleration	Here zero force doesn't correspond to zero acceleration
Examples : A space shuttle moving with constant velocity relative to the earth, any reference frame that is not accelerating, a reference frame attached to Earth	**Examples :** Elevator, Rotating frames, any accelerating frames

Galilean relativity

- If Newton's laws are true in any reference frame, they are also true in any other frame moving at constant velocity with respect to the first one.
- *Any two observers moving at constant speed and direction with respect to one another will obtain the same results for all mechanical experiments.*
- A ball thrown up by a passenger in an aero plane moving with constant velocity, observes the ball moving in a vertical path. The motion of the ball is precisely the same if it is thrown while at rest on the Earth.
- For an observer on earth, the ball thrown by the passenger is seen as parabolic. Both observes agree with respect to the laws of physics.
- The law of gravity and the equations of motion under constant acceleration are obeyed.
- **There is no preferred frame of reference of describing the laws of mechanics. The fundamental physical laws and principles are identical in all inertial frames.**

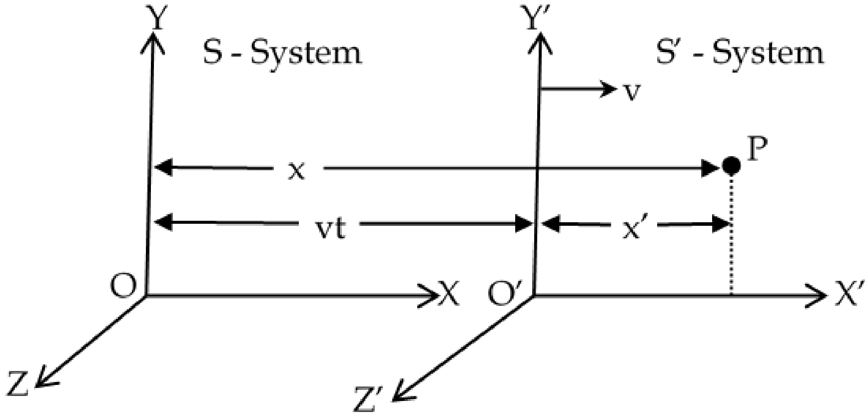
Galilean Transformation

- A set of equations that relate the space and time coordinates of two systems moving at a constant velocity relative to each other.
- They are adequate to describe phenomena at speeds much smaller than the speed of light.
- Galilean transformations formally express the ideas that space and time are absolute; that length, time, and mass are independent of the relative motion of the observer.

Galilean transformation equations

S and S' are two inertial frames of reference. S is at rest and S' moving with a uniform velocity v with respect to S along the positive X - direction. Also $v \ll c$ where c is the speed of light.

At t = 0, the origins of the two frames coincide. An event occurs at a point P. The observer O in frame S determines the position of the event as coordinates x, y, z . The observer O' in frame S' determines the position of the event as coordinates $x^\wedge{}', y^\wedge{}', z'$.



The time is assumed to proceed at the same rate in both the frames.
At a later instant of time, the distance between the two frames is $\boldsymbol{vt}$. From the diagram,
$OP = O'P + OO'$ ...(1)
As $OP = x, \quad O'P = x' \quad and\; OO' = vt$,
the relation $OP = O'P + OO'$ is $x = x' + vt$ or $x' = x - vt$
As there is no relative motion between S and S' along Y and Z directions, $y = y'$ and z $= z'$.
The transformation equations given from S to S' are
$\boldsymbol{x' = x - vt}$ **,** $\boldsymbol{y' = y}$ **,** $\boldsymbol{z' = z}$ **and** $\boldsymbol{t' = t}$ **......(2)**
These are called Galilean position transformation equations
The inverse transformation from S' to S are given by
$x = x' + vt$, $y' = y$, $z' = z$ and $t' = t$
Differentiating the following eqn. (2) with respect to time t,
$\boldsymbol{x' = x - vt}$ **,** $\boldsymbol{y' = y}$ **,** $\boldsymbol{z' = z}$ and $\boldsymbol{t' = t}$
$\frac{dx'}{dt} = \frac{dx}{dt} - v$, $\frac{dy'}{dt} = \frac{dy}{dt}$ and $\frac{dz'}{dt} = \frac{dz}{dt}$ ....(3)
Let $\frac{dx'}{dt} = u'$ be the velocity of the particle at P as measured by S',
$\frac{dx}{dt} = u$ be the velocity of the particle at P as measured by S,
the equations are $\boldsymbol{u' = u - v}$ **,** $\boldsymbol{\frac{dy'}{dt} = \frac{dy}{dt}}$ and $\boldsymbol{\frac{dz'}{dt} = \frac{dz}{dt}}$ ....(4)
These equations are called non relativistic ($v \ll c$) **Galilean velocity transformation equations.**
The equation $\boldsymbol{u' = u - v}$ can be expressed as $\boldsymbol{u = u' + v}$
i.e., the velocity of a particle as measured by a stationary frame is equal to the sum of the velocity of this particle as measured from the moving frame and velocity of S' frame with respect to S frame.
If $\boldsymbol{u = 0}$, i.e. the particle is at rest with respect to S frame, then $\boldsymbol{u' = -v}$.
The particle appears to move with uniform velocity in S' frame.
Thus the Newton's first law is obeyed in both the frames.

Differentiating these relations again $\frac{dx\prime}{dt} = \frac{dx}{dt} - v$, $\frac{dy\prime}{dt} = \frac{dy}{dt}$ and $\frac{dz\prime}{dt} = \frac{dz}{dt}$

$\frac{d^2x\prime}{dt^2} = \frac{d^2x}{dt^2}$, $\frac{d^2y\prime}{dt^2} = \frac{d^2y}{dt^2}$, $\frac{d^2z\prime}{dt^2} = \frac{d^2z}{dt^2}$. ($\boldsymbol{v}$ is a constant).

As $\frac{d^2x\prime}{dt^2} = a'$ is the acceleration of the particle as measured by S' frame and $\frac{d^2x}{dt^2} = a$ is the acceleration in S frame.

Thus $\boldsymbol{a'} = \boldsymbol{a}$ ....(5) Thus the acceleration of the particle as measured by the two frames are the same or **acceleration is invariant.** Equation (5) is the **Galilean acceleration transformation equation.**

Multiplying eqn. (5) by m, the mass of the particle, $\boldsymbol{ma'} = \boldsymbol{ma}$ or $\boldsymbol{F'} = \boldsymbol{F}$.

Thus the Newton's laws are valid in S' frame also and thus a inertial frame.

Thus the laws of mechanics are same in all inertial frames of reference which is the principle of Galilean relativity.

FRICTION

When a solid body moves on a surface, an opposing force acts, which resists the forward motion of the body.

The resistive force, which opposes the forward force applied on the body due to relative motion between the two surfaces in contact is known as the **force of friction or frictional force.**

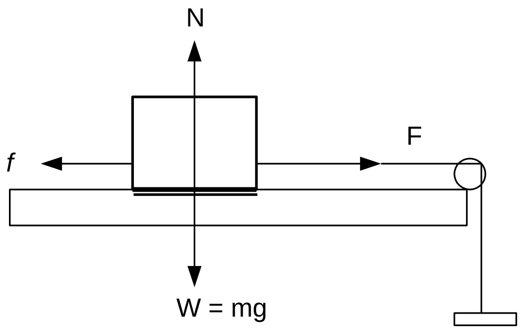
The magnitude of the frictional force depends on the nature of the two surfaces in contact. This force always act in the direction opposite to that of the motion of the body.

Examples of friction - We are able to hold a pen and write due to friction. We are able to walk and run on a road, without slipping, due to the friction offered by the ground.

Explanation of friction: Consider a wooden block kept on the horizontal surface. One end of a string is attached to the block and the free end of the string moves on a pulley to a scale pan, as shown.

When weights are gradually added to the scale pan, the wooden block does not move initially. This is due to the applied horizontal force F opposed by an equal force f due to friction. This is called as the ***force of static friction**.

When weights are further (increase of F) increased, the frictional force also increases and the block still remains static. Therefore force of static friction is **self-adjusting force**.



As the weights are added further, at a particular value of weight, the wooden block would just begin to move. The minimum force F required to just move the block is known as **limiting force of friction.**

The limiting force of friction is equal to the **maximum force of static friction f_s.**
Once the block starts moving.it gets accelerated due to the force F and thus the force of friction decreases.
The minimum force F just required to maintain the uniform motion of the block is called the ***force of kinetic friction_(or force of sliding friction) (f_k)**.
If the wooden block is provided with wheels, the block starts rolling on the surface.
The minimum force F required to pull the block. provided with wheels, is equal to the **force of rolling friction (f_r),** developed between the two surfaces.
It is found that $f_r < f_k < f_s$..
The maximum force of static friction between any two surfaces is (a) independent of the area of contact and (b) proportional to be normal reaction N

The normal force N is exerted by the surface on the body, perpendicular to the surfaces in contact. It is equal and opposite to the weight W of the wooden block.

Coefficient of static friction The ratio of the magnitude of the limiting force of static friction (f_s) to the magnitude of the normal force N is called the coefficient of static friction (μ_s) It is a constant. $\mu_s = \frac{f_s}{N}$

Coefficient of kinetic friction_Kinetic friction arises between two surfaces when they are in relative motion. The ratio of the magnitude of the force of kinetic friction f_k to the magnitude of the normal force N is called the coefficient of kinetic friction and is found to be a constant, $\mu_k = \frac{f_k}{N}$

Coefficient of rolling friction_When a body rolls over a surface, the force of friction developed between the two surfaces is known as the force of rolling friction, f_r
The ratio of the magnitude of the force of rolling friction f_r to the magnitude of the normal force N is called the coefficient of rolling friction and is a constant. i.e., $\mu_r = \frac{f_r}{N}$

It is found that $\mu_r < \mu_k < \mu_s$ *as* $f_r < f_k < f_s$.

Laws of static and kinetic friction

1 Limiting friction depends on nature of surface in contact.

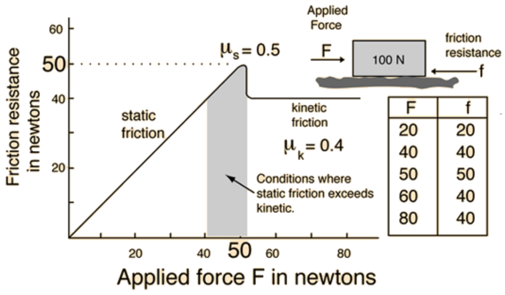
2. It acts tangential to the two surfaces in contact and is opposite to the direction of motion of the body.

3. The force of friction between two surfaces opposes their relative motion.

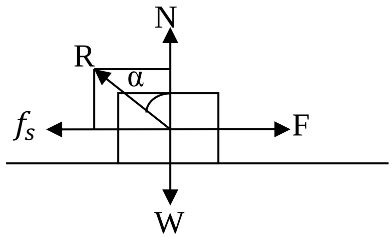
4.The force of friction is independent of the area of contact of the given surfaces when the normal force is constant.

5. The force of limiting friction (static or kinetic) is proportional to the total normal force. In the case of kinetic friction, it is independent of the velocity of the surfaces.

The graph shows the variation of frictional resistance with applied force.

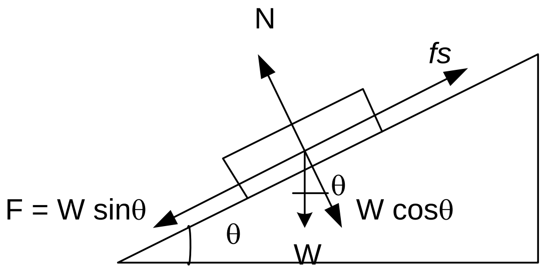


Angle of friction



Consider an object placed on a horizontal surface as shown above. W is the weight and N is the normal force or the normal reaction. f_s is the limiting force of friction and F is the applied force on the object. The angle between the normal reaction and the resultant of f_s and N (i.e. R) is called the angle of friction α. From the diagram the angle of contact is tan α = f_s/N. As f_s/N = μ_s coefficient of friction. Thus **tan α = μ_s**

Expression for angle of contact or angle of repose



Consider a wooden block placed on an inclined plane which is slanting at an angle θ with the horizontal. The forces acting on the block are

(a) W = mg. the weight of the block acting vertically downwards.

(b) The normal force N exerted by the plane on the Wooden block and

(c) the force of static friction $f_s$opposite to the impending motion of the block exerted by the plane on the body.
When the block is at rest, the condition is N + f_s + W =0

The force W can be resolved into two components i.e..Wsinθ and Wcosθ as shown in the diagram. Then the net force on the block along the vertical direction is N - Wcosθ = 0 or ***N = Wcosθ* ....(1)**

and along the direction of impending motionf_s - Wsinθ = 0or ***f_s = Wsinθ* ....(2)**

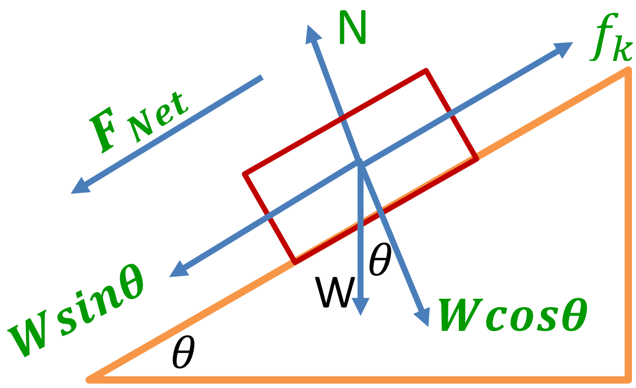
Let the angle of inclination θ between the planes be slowly increased until the block just starts slipping down the inclined plane. This angle of inclination, θ = θ_s is called as the **angle of repose or critical angle or angle of friction.**

By definition $\mu_s = \frac{f_s}{N}$ ...(3) Substituting for f_s and N from (1) and (2) in (3) we get

$\mu_s = \frac{W\sin\theta}{W\cos\theta} = \tan\theta$ **or $\mu_s = \tan\theta$ or $\theta = \tan^{-1}\mu_s$**

Acceleration of a body sliding down an inclined plane (With friction)

Consider a block of mass m sliding down an inclined plane with an acceleration a which is inclined at an angle greater than the angle of contact.



The net force $\boldsymbol{F}$ acting on the body along the incline is $\boldsymbol{F_{Net} = W\,sin\theta - f_k}$ ...(1)

As $\boldsymbol{f_k = \mu_k N}$ ..(2) also $\boldsymbol{N = W\,cos\theta}$ ...(3)

Eqn. (3) in (2) gives $f_k = \mu_k W\,cos\theta$ ...(4)

Eqn. (4) in (1) $\boldsymbol{F_{Net}} = W\,sin\theta - \mu_k W\,cos\theta$ ...(5)

As $\boldsymbol{F_{Net} = ma}$ *and* $\boldsymbol{W = mg}$ Eqn. (5) is $ma = mg\,sin\theta - \mu_k mg\,cos\theta$

The acceleration is $\boldsymbol{a = g\,sin\theta - \mu_k g\,cos\theta}$

If l is the length of inclined plane, the distance travelled by the block is $l = \frac{1}{2}at^2$ $(As\ \ s = ut + \frac{1}{2}at^2\ \ here\ \ s = l,\ initial\ velocity\ u = 0)$

Thus the time taken by the block to travel this distance is

$t = \sqrt{\frac{2l}{a}}$ or $t = \sqrt{\frac{2l}{g\,sin\theta - \mu_k g\,cos\theta}}$

Acceleration of a body sliding down an inclined plane (Without friction)

For a body sliding along the incline, the net force $\boldsymbol{F}$ acting on the body in the absence of friction is $F_{Net} = W\,sin\theta$ ...(1)

As $F_{Net} = ma$ *and* $W = mg$

Eqn. (1) is $ma = mg\,sin\theta$ The acceleration is $\boldsymbol{a = g\,sin\theta}$

If l is the length of inclined plane, the distance travelled by the block is $l = \frac{1}{2}at^2$ $(As\ \ s = ut + \frac{1}{2}at^2\ \ here\ \ s = l,\ initial\ velocity\ u = 0)$

Thus the time taken by the block to travel this distance is

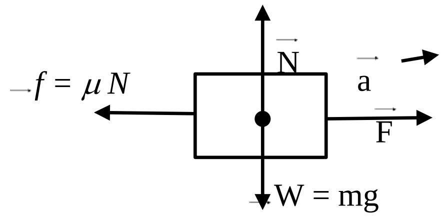
$t = \sqrt{\frac{2l}{a}}$ or $t = \sqrt{\frac{2l}{g\,sin\theta}}$

Free body diagram

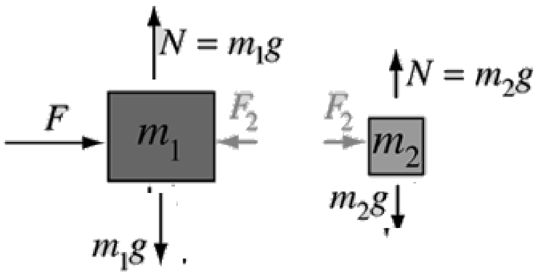
Free-body diagrams are diagrams used to show the relative magnitude and direction of all forces acting upon an object in a given situation. The size of the arrow in a free-body diagram shows the magnitude of the force. The direction of the arrow shows the direction that the force is acting. Each force arrow in the diagram is labeled to indicate the exact type of force. It is generally customary in a free-body diagram to represent the object by a box and to draw the force arrow from the center of the box outward in the direction that the force is acting.

Examples of free body diagram

1. Consider a body experiencing a force F moving on a surface with constant acceleration. The free body diagram is as shown with all the forces acting ob the object. $F_{net} = F - f = ma$. From this equation acceleration of the body can be determined. Also, frictional force $= \mu N = \mu mg$. $(W = N)$



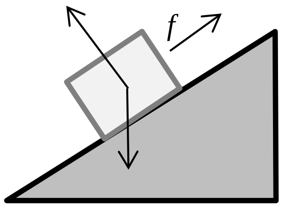
2. **Consider two bodies in contact** experiencing force F as shown. Since F is the only net force acting on the two masses, it determines the acceleration of both. The free body diagram for both the masses are as shown. The acceleration is given by $a = \frac{F}{m_1 + m_2}$. If frictional force is acting then, $a = \frac{F-f}{m_1 + m_2}$ .where $f = \mu(m_1 + m_2)$ The force on the smaller mass is $F_2 = m_2 a$. From Newton's third law, F_2 also acts on mass m_1 . Thus net force on $m_1$is $F_1 = F - F_2 = (m_1 + m_2)a - m_2 a = m_1 a$.

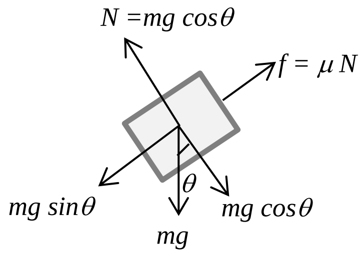


3. **Consider a set of three masses connected by strings** as shown along with the free body diagrams for each mass. . The force F is the only net force acting on the system of

three masses, which are constrained to accelerate together. Therefore Newton's 2nd law gives the acceleration. $a = \frac{F}{m_1 + m_2 + m_3}$. The equations corresponding to each mass as shown in the free body diagram are $T_1 = m_1 a$, $T_2 - T_1 = m_2 a$ and $F - T_2 = m_3 a$. From these equations the tension in the string can be calculated.

4. Consider a body of mass m sliding down an inclined plane as shown. The corresponding free body diagram is represented as shown with the forces after resolving mg into its components. Assume frictional force is acting between the body and the surface of the inclined plane.





From the diagram $N = mg\,cos\theta$ and the net force acting on the body is

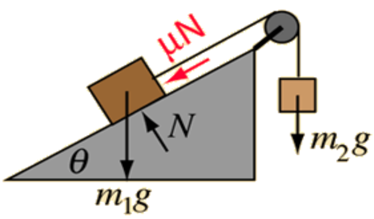
$F = mg\,sin\theta - f$ As $F = ma$ and $f = \mu N = \mu mg\,cos\theta$

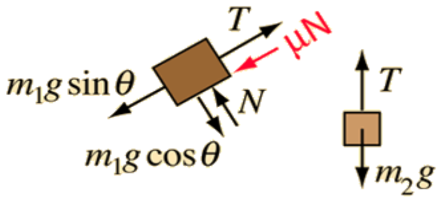
Thus the acceleration of the body sliding down an inclined plane can be found by equation $ma = mg\,sin\theta - \mu mg\,cos\theta$.

In the absence of friction this equation can be written as $ma = mg\,sin\theta$.

5. A body moving on an inclined plane attached to a string moving over a pulley

Consider a body of mass m_1 fixed to a string moving on an inclined plane attached to a string which moves on a pulley. The other end of the string is attached to a bosy of mass m_2 as shown. The free body diagram is as shown.





The mass m_2 is larger than mass m_1. Thus the m_1 moves up the plane and mass m_2 moves down with an acceleration a. From Newton's second law for the mass m_2, the equation is $m_2 g - T = m_2 a$ ....(1)

For the mass m_1, the equation is $T - m_1 g\,sin\theta = m_1 a$ ....(2)

Thus $T = m_1 g\,sin\theta + m_1 a$ ....(3)

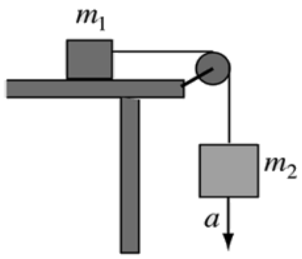
Substituting for T from (3) in (1), we get $m_2 g - (m_1 g\,sin\theta + m_1 a) = m_2 a$

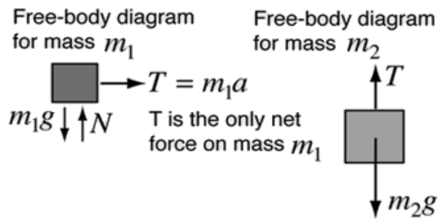
Simplifying for a we get $a = \frac{m_2 g - m_1 g\,sin\theta}{m_1 + m_2}$

In the presence of friction equation (2) becomes

$T - m_1 g\,sin\theta - \mu N = m_1 a$ where $N = m_1 g\,cos\theta$.

6. Consider a body of mass m_1 moving on a horizontal plane with a string attached it which moves on a pulley. The other end of the string is attached to mass m_2 as shown. The free body diagram for the two masses are as shown.





Applying Newton's second law to the mass m_1, we have $T = m_1 a$ ....(1)

For the mass m_2, we have $m_2 g - T = m_2 a$ ....(2)

Substituting for T from (1) in (2) we get $m_2 g - m_1 a = m_2 a$ or $a = \frac{m_2 g}{m_1 + m_2}$.

Atwood machine

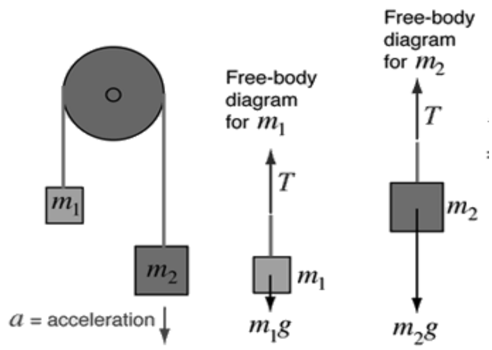
The **Atwood machine** was invented in 1784 by the English mathematician **George Atwood** as a laboratory experiment to verify the mechanical laws of motion with constant acceleration and to measure **acceleration due to gravity**.

An Atwood machine consists of two objects of mass m_1 and m_2, connected by an inextensible massless string over an massless pulley.

When the two objects on the machine are of **equal masses**, then the system will be in **neutral equilibrium** and no motion of any kind will take place.

If there is a **very small difference in their masses**, then their **acceleration will be small and can be easily measured.** This is what makes the Atwood machine quite useful to determine acceleration due to gravity (g).

Consider an Atwood machine as shown. Let the pulley be frictionless and pulley mass is assumed to be negligible. Let the two masses be m_1 and m_2 with m_2 greater than m_1. Using Newton's second law, for the mass m_1 the equation is



$F_{net} = T - m_1 g$ As $F_{net} = m_1 a$

Thus $m_1 a = T - m_1 g$ ....(1)

or $T = m_1 a + m_1 g$ ....(2)

For the mass m_2, $F_{net} = m_2 g - T$

or $m_2 a = m_2 g - T$ ....(3)

Substituting for T from (2) in (3), we get $m_2 a = m_2 g - (m_1 a + m_1 g)$

Simplifying the above equation we get $a = \frac{(m_2 - m_1) g}{m_1 + m_2}$.

or acceleration due to gravity $g = \frac{(\boldsymbol{m_1} + \boldsymbol{m_2})}{(\boldsymbol{m_2} - \boldsymbol{m_1})} \boldsymbol{a}$

The tension in the string is found by substituting $\boldsymbol{a} = \frac{(\boldsymbol{m_2} - \boldsymbol{m_1})}{(\boldsymbol{m_1} + \boldsymbol{m_2})} \boldsymbol{g}$ in equation (3) or (6)

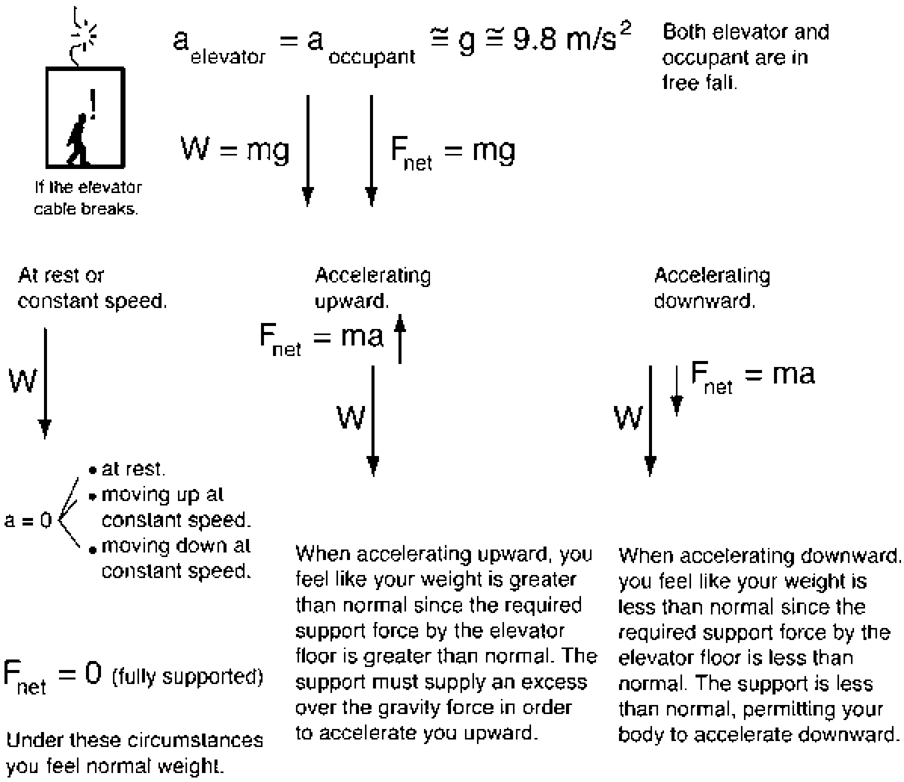
Equation (3) is $\boldsymbol{T} = \boldsymbol{m_1 a} + \boldsymbol{m_1 g}$

$$T = m_1\left(\frac{(m_2-m_1)}{m_1+m_2}\boldsymbol{g}\right) + \boldsymbol{m_1 g} = m_1\left(\frac{m_2-m_1}{m_1+m_2} + \mathbf{1}\right)g$$

$$T = m_1\left(\frac{m_2-m_1+m_1+m_2}{m_1+m_2}\right)g = m_1\left(\frac{2\,m_2}{m_1+m_2}\right)g$$

or Tension in the string is $T = \left(\frac{2m_1\,m_2}{m_1+m_2}\right)g$

Elevator with an occupant in it. Different cases are explained below. (1) In the first case, if the moving elevator cable breaks we feel weightless. (2) If the elevator is at rest or moving with constant speed, we feel normal weight. (3) When the elevator accelerates upwards we feel more weight and when it is accelerating downwards, we fell less weight.



Motion in a resistive medium :

When a body moves through a fluid, it experiences frictional force (viscous force) which opposes its motion. The medium like liquid or a gas that opposes motion of a body through it called resistive medium.

Drag Force (sometimes called **air resistance** or **fluid resistance**) is the opposing force acting on a body opposite to its direction of motion due to the relative motion between the body and the medium.

This can exist between two fluid layers (or surfaces) or a fluid and a solid surface. Drag forces depend on velocity. They are of two categories:

(1) Drag force is proportional to the velocity for a laminar flow or a streamline flow i.e. for objects moving with lesser velocities. **(** $\boldsymbol{F_D \propto v}$ ***or*** $\boldsymbol{F_D = -cv}$ **).** This is applicable to a body moving through a more viscous medium like liquids.

(2) The drag force is proportional to square of the velocity for a turbulent flow i.e. for objects moving with higher velocities **($F_D \propto v^2$ or $F_D = -cv^2$).** This is applicable to bodies moving through less viscous medium like air or any other gaseous medium. These velocity dependent forces are non conservative forces.

In the above expressions c is called the **drag coefficient**. It depends on the viscosity and density of the medium. It also depends on the effective area of the surface of the body in contact with the fluid**.**

Examples : (i) The motion of a parachut is retarded as it drifts down towards earth (ii) The motion of a cannon ball sinking in the ocean is retarded. (iii) motion of a solid object through a viscous liquid like castor oil in which the object attains constant velocity.

Terminal velocity

The maximum constant velocity attained by the body moving under gravity in a resistive medium when the drag force equals the weight of the body is called the **terminal velocity denoted by v_t**.

Explanation : When a body is dropped in a resistive medium like a liquid, it experiences two forces as it comes down. They are (1) weight of the body $W = mg$ acting vertically downwards and (2) the drag force which acts vertically upwards. With time, the resistive force increases and the acceleration decreases. The acceleration becomes zero when the magnitude of the resistive force becomes equal to the weight of the object. At this point the object will attain the maximum velocity.

Expressions for terminal velocity

1. Consider the case of **drag force proportional to the velocity**. The drag force is $F_D = -cv$ and the weight is $W = mg$. When the two forces are equal, $v = v_t$. Thus $cv_t = mg$ or $\boldsymbol{v_t = \frac{mg}{c}}$. Thus v_t depends on the mass of the body and the drag coefficient.
2. Consider the case of **drag force proportional to square of the velocity**. In this case $cv_t^2 = mg$ or $v_t^2 = \frac{mg}{c}$ or $\boldsymbol{v_t = \sqrt{\frac{mg}{c}}}$.

Motion of an object under the action of gravity in a resistive medium with the drag force proportional to the velocity

Consider a spherical object of mass m falling under the action of gravity in a resistive medium in which resistive force is proportional to the velocity of the object. The two forces acting on the body are the force of gravity $F_g = mg$ and the resistive force or the drag force $F_D = -cv$.where is the resistive force per unit velocity.

To find the velocity of the object :

Applying the Newton's second law to this vertical motion by considering vertically downward motion to be positive (Y – direction)

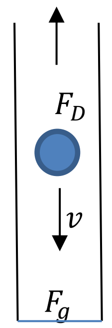
The resultant force on the sphere is $\sum F_y = mg - cv$

As $\sum F_y = ma$, the above equation becomes $ma = mg - cv$ …..(1) or $a = g - \frac{c}{m}v$

At the instant when the object is released, i.e. $t = 0, v = 0.$

Thus from the above equation, $a = g$.

When the body attains terminal velocity i.e. at $v = v_t$, a = 0.



Thus $\sum F_y = ma = 0$.Thus $a = g - \frac{c}{m}v_t = 0$.

The expression for terminal velocity is $\boldsymbol{v_t = \frac{mg}{c}}$ ……(2)

Now as $a = \frac{dv}{dt}$, equation (1) can be written as $m\frac{dv}{dt} = mg - cv$

Dividing the above equation throughout by cv , we get $\frac{m}{cv}\frac{dv}{dt} = \frac{mg}{cv} - 1$

Using equation (2) in the above equation, i.e. $\boldsymbol{\frac{mg}{c} = v_t}$

$\frac{m}{cv}\frac{dv}{dt} = \frac{v_t}{v} - 1$ or $\frac{m}{cv}\frac{dv}{dt} = \frac{v_t}{v} - 1$

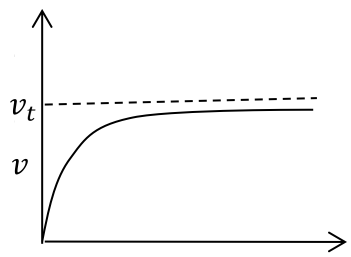
Simplifying this equation $\frac{m}{cv}\frac{dv}{dt} = \frac{v_t - v}{v}$ or $\frac{dv}{(v_t - v)} = \frac{c}{m}dt$

Integrating, $\int_0^v \frac{dv}{(v_t - v)} = \int_0^t \frac{c}{m}dt$ we get $-log(v_t - v)\Big|_0^v = \frac{c}{m}t$

$-log(v_t - v) - (-\log v_t) = \frac{c}{m}t$ or $log(v_t - v) - \log v_t = -\frac{c}{m}t$

or $\frac{v_t - v}{v_t} = e^{-\frac{c}{m}t}$ or $1 - \frac{v}{v_t} = e^{-\frac{c}{m}t}$ or $\frac{v}{v_t} = 1 - e^{-\frac{c}{m}t}$

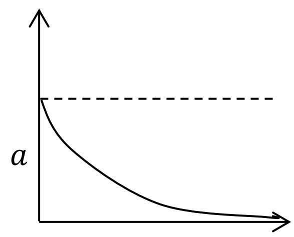
Thus $\boldsymbol{v = v_t\left(1 - e^{-\frac{c}{m}t}\right)}$ ……(3)



Equation (3) gives the expression for the velocity of the object under gravity in the resistive medium.

The variation of v versus t is as shown in the graph. As time increases the velocity increases and reaches maximum value corresponding to the terminal velocity when acceleration becomes zero.

When $t = \tau = \frac{m}{c}$, equation (3) becomes $v = v_t(1 - e^{-1}) = v_t\left(1 - \frac{1}{e}\right) = v_t\left(1 - \frac{1}{2.718}\right) = 0.632\, v_t$. Thus τ is the time constant defined as the time required by the object to reach 63.2% of the terminal velocity.



To find the acceleration of the object :

Differentiating equation (3), we get

$\frac{dv}{dt} = \frac{d}{dt}\left[v_t\left(1 - e^{-\frac{c}{m}t}\right)\right] = v_t\left(0 - \left(-\frac{c}{m}\right)e^{-\frac{c}{m}t}\right)$

As $a = \frac{dv}{dt}$, $a = v_t\left(\frac{c}{m}e^{-\frac{c}{m}t}\right)$ From eqn. (2) $v_t = \frac{mg}{c}$

Substituting for v_t in the above equation $a = \frac{mg}{c}\left(\frac{c}{m}e^{-\frac{c}{m}t}\right) = ge^{-\frac{c}{m}t}$

Thus $\boldsymbol{a = ge^{-\frac{c}{m}t}}$ This equation shows that when t = 0, a = g and as t tends to infinity the acceleration becomes zero as shown in the graph.

To find the displacement of the object :

The velocity of the object is given by $v = \frac{dy}{dt}$ or $dy = v\,dt$

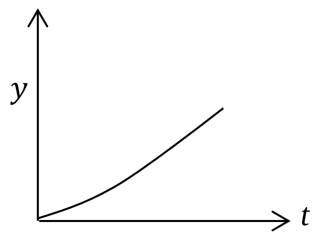
Integrating

$y = \int_0^t v\,dt = \int_0^t v_t\left(1 - e^{-\frac{c}{m}t}\right) dt = v_t t - v_t\left(-\frac{m}{c}e^{-\frac{c}{m}t}\right)_0^t$

or $y = v_t t + v_t\frac{m}{c}\left(e^{-\frac{c}{m}t} - e^0\right) = v_t t + v_t\frac{m}{c}\left(e^{-\frac{c}{m}t} - 1\right)$

or $\boldsymbol{y = v_t\left[t - \frac{m}{c}\left(1 - e^{-\frac{c}{m}t}\right)\right]}$

The variation of displacement with time is as shown.



Motion in case of quadratic resistance – Drag force proportional to square of the velocity

For an object falling under gravity at a high speed in a resistive medium the drag force is proportional to the square of the velocity. It is given by $F_D \propto v^2$ or $F_D = -Dv^2$

The constant D is given by $D = \frac{1}{2}C\,\rho A$ where A is the effective cross-sectional area of the falling object, ρ is the density of the surrounding resistive medium and C is the drag coefficient that depends on the shape of the moving object and whose value generally lies in the range 0.5 - 1.0.

Thus the magnitude of the drag force acting on the object is given by

$F_D = -\frac{1}{2}C\,\rho A v^2$.

The net force acting on the object falling under gravity is $\sum F = F_g - F_D$.

As $\sum F = ma$ and $F_g = mg$, the above equation can be written as

$ma = mg - \frac{1}{2}C\,\rho A v^2$ or $a = g - \frac{C\,\rho A v^2}{2m}$

By definition of terminal velocity, when $v = v_t a = 0$,

Then the equation $a = g - \frac{C\,\rho A v^2}{2m}$ becomes

$0 = g - \frac{C\,\rho A v_t^2}{2m}$ or $mg = \frac{C\,\rho A v_t^2}{2}$ or $v_t^2 = \frac{2mg}{C\rho A}$.

The expression for the terminal velocity is $\boldsymbol{v_t = \sqrt{\frac{2mg}{C\rho A}}}$. For example, a sky diver of mass 75 kg with cross sectional area 0.7 m^2, the terminal velocity is 60 ms^{-1}.

Numerical problems

1. The driver of a car moving at a speed of 20 m s^{-1} finds a child on the road 50 m ahead. He stops the car 10 m behind the child. If the mass of the car is 1000 kg, calculate the average force on the car and the time taken to stop the car.

Given data : $u = 20ms^{-1}$, $s = 50 - 10 = 40m$, $v = 0$, $m = 1000\ kg$

Formulae : $v^2 - u^2 = 2as$ $F = ma$ and $v = u + at$

Solution : $a = \frac{v^2 - u^2}{2\,s} = \frac{0 - 20^2}{2\times 40} = -5ms^{-2}$

$F = ma = 1000 \times 5 = 5000\ N$

$t = \frac{v-u}{a} = \frac{0-20}{-5} = 4\ s$

2. If a body is moving with a speed of 15 ms^{-1} and the coefficient of friction between the ground and the body is 0.3, find the distance travelled by the body before it comes to rest. $g = 9.8\ \mathrm{m\,s^{-2}}$

Given data : u = 15 ms^{-1}, $v = 0$, $\mu_k = 0.3$, $g = 10\ \mathrm{m\,s^{-2}}$

Formula : $F = ma$, $(f_k = \mu_k N = \mu_k W)$ $W = mg$ and $v^2 - u^2 = 2as$

Solution : $F = ma = \mu_k W = \mu_k mg$ or $ma = \mu_k mg$ $a = \mu_k g$

$a = 0.3 \times 9.8 = 2.94\ N$. $s = \frac{v^2-u^2}{2a} = \frac{0-15^2}{2\times 2.94} = 38.26\ m$ $s = 38.26\ m$

3. A block of wood of mass 5 kg placed on an inclined plane moves down the plane with a constant speed when the angle of inclination is 20^0. Calculate the force of sliding friction between the block and the surface of the plane. Given the acceleration due to gravity is $g = 9.8\ \mathrm{m\,s^{-2}}$.

Given data : $m = 5\ kg$, $\theta = 20^0$, $f_k = ?$ $g = 9.8\ \mathrm{m\,s^{-2}}$

Formula :$f_k = m\ g\ sin\theta$

Solution : $f_k = 5 \times 9.8 \times sin20 = 5 \times 9.8 \times 0.342 = 16.758\ N$ $f_k = 16.758\ N$

4. A force of 10 N acts on a body of mass 1 kg lying on a table with the coefficient of static friction as 0.2. Calculate the normal force, the net force and the acceleration of the body.

Given data : $F = 10\ N$, $m = 1\ kg$, $\mu_s = 0.2$, $N = ?$ $F_{net} = ?$, $a = ?$

Formula : $[N = mg,\ \ F_{net} = F - f\ \ where\ f = \mu N,\ F_{net} = ma\ ]$

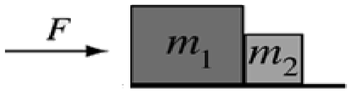
Solution : $N = 1 \times 9.8 = 9.8ms^{-2}$

$F_{net} = F - f = 10 - (0.2 \times 9.8) = 8.04N$ $a = \frac{F_{net}}{m} = 8.04\ ms^{-2}$

5. Two bodies of masses $m_1 = 3\ kg$ and $m_2 = 2\ kg$ which are in contact on a frictionless table as shown. A force of 10 N acts on the mass m_1 as shown. Find the acceleration produced in the bodies and also the contact force.

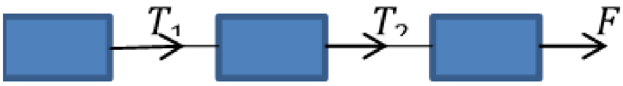
Given data :$m_1 = 3\,kg, m_2 = 2\,kg, \quad F = 10\,N, \quad a = \ ?\ F_c = ?$

Formula : $a = \frac{F}{m_1 + m_2}$ and $F_c = m_2 a$



Solution : $a = \frac{10}{3+2} = 2\,ms^{-2}$ $\quad F_C = m_2 a = 2 \times 2 = 4\,N$

6. Three blocks of the same mass $m = 1kg$ each, connected by cords are pulled by a force F on a smooth horizontal surface as shown. Find the tensions T_1, T_2 and the force F if the acceleration is 10 ms^{-2}



Data given : $m = 1kg,$

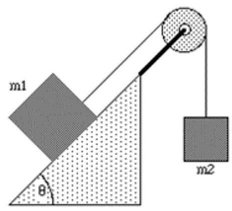
$a = 10ms^{-2}, \quad T_1 = \ ? \quad T_2 = \ ? \quad F = \ ?$

Formula : $T_1 = ma, \quad T_2 - T_1 = ma, \quad F - T_2 = ma$

Solution : $T_1 = 1 \times 10 = 10N, T_2 = ma + T_1 = 10 + 10 = 20\,N$

$F = ma + T_2 = 10 + 20 = 30\,N$

7. Two blocks are connected over a massless pulley as shown. The mass of block 1 is 10 kg and the coefficient of kinetic friction is 0.2. Block A slides down the incline at constant speed. Find the mass of block 2. $(\theta = 30^0)$



Data given : $m_1 = 10\,kg, \quad \mu_k = 0.2, \theta = 30^0, m_2 = \ ?$

Formula : $f_k = \mu_k N = \mu_k mgcos\theta, \quad T = m_2 g$

$constant\ speed\ means\ a = 0,\ No\ net\ force$

$m_1 gsin\theta - T - f_k = 0$

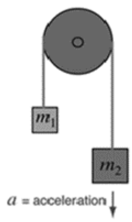
Solution : $T = m_1 gsin\theta - f_k = m_1 gsin\theta - \mu_k mgcos\theta$

$T = 10 \times 9.8 \times sin30 - 0.2 \times 10 \times 9.8 \times cos30 = 49 - 16.95 = 32.05N$

$m_2 = \frac{T}{g} = \frac{32.05}{9.8} = 3.27\,kg$

8. Find the acceleration of the system and the tension in the string of the diagram shown. ($m_1 = 2\,kg,\ m_2 = 8\,kg,$ g = 10 ms^{-2})

Given data : $m_1 = 2\,kg,\ m_2 = 8\,kg,$ g = 10 ms^{-2}



Formula : $a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g, \quad T = m_1 a + m_1 g$

Solution : $a = \frac{8-2}{8+2} \times 10 = 6\,ms^{-2}$

$T = 2 \times 6 + 2 \times 10 = 32\,N$

9. A block of mass $m_1 = 3\,kg$ is resting on a smooth horizontal surface. It is connected to a mass $m_2 = 2\,kg$ by a massless string passing over a pulley. Find the acceleration of the system and the tension in the string.

Data given : $m_1 = 3\,kg, m_2 = 2\,kg, \quad a = ?\ \ T = ?$

Formula : $a = \left(\frac{m_2}{m_1+m_2}\right) g$ and $T = m_1 a$

($T = m_1 a$ For mass m2, $m_2 g - T = m_2 a$ or $m_2 g - m_1 a = m_2 a$)

Solution : $a = \frac{2}{3+2} \times 9.8 = 3.92\, ms^{-2}$

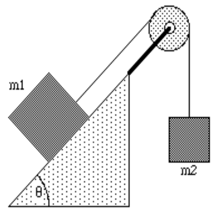
$T = 3 \times 3.92 = 11.76$ N

10. Find the acceleration of a block 1 and tension in the string shown in the diagram if the coefficient of kinetic friction is 0.2. Given $m_1 = 5\,kg,\ m_2 = 10\,kg,\ g = 10\,ms^{-2},\ \theta = 30^0$.

Given data : $m_1 = 5\,kg,\ m_2 = 10\,kg,\ g = 10\,ms^{-2},\ \theta = 30^0,\quad \mu_k = 0.2\ .\ a\ =?$

Formula : *For block* 1 $m_1 a = T - \mu_k N - m_1 g sin\theta$ and $N = m_1 g cos\theta$

For block 2, $m_2 a = m_2 g - T,$ or $T = m_2 g - m_2 a$



Thus $m_1 a = (m_2 g - m_2 a) - \mu_k N - m_1 g sin\theta$

Solution : $a = \frac{m_2 g - \mu_k N - m_1 g sin\theta}{m_1 + m_2}$ or $a = \frac{m_2 g - \mu_k m_1 g cos\theta - m_1 g sin\theta}{m_1 + m_2}$

$a = \frac{10\times 9.8 - 0.2\times 5\times cos30 - 5\times 9.8\times sin30}{5+10} = 4.84 ms^{-2}$

$T = m_2 g - m_2 a = 10 \times 9.8 - 10 \times 4.84 = 49.6\, N$

11. A trolley of mass 1.5 kg is pulled along a table by a string attached to a mass of 2 kg hanging down over pulley. If the trolley starts from rest, calculate the distance travelled by it in 5 s.

Given data : $m_1 = 1.5\,kg,\ m_2 = 2\,kg,\quad u = 0,\ \ t = 5\,s,\quad s\ =?$

Formula : $a = \left(\frac{m_2}{m_1+m_2}\right) g,$ *and* $s = ut + \frac{1}{2} a t^2$

Solution : $a = \frac{2}{1.5+2} \times 9.8 = \mathbf{5.6\ ms^{-2}}$

$s = 0 + \frac{1}{2} \times 5.6 \times 5^2 = \mathbf{70\ m}$

12. Two bodies A and B of masses 1 kg and 0.25 kg respectively are connected by a light string passing over smooth pulley at the top of smooth inclined plane. The system remains at rest when A lies on the inclined plane and B hangs vertically down. Calculate the angle of inclination of inclined plane and force exerted by the inclined plane on the body A.

Given data : $m_1 = 1\,kg,\quad m_2 = 0.25\,kg,\quad \theta = ?\quad F = ?$

Formula : $m_1 g sin\theta = T,$ *and* $T = m_2 g$

Solution : $T = m_2 g = 0.25 \times 9.8 = 2.45\,N$

$sin\theta = \frac{T}{m_1 g}$ or $\theta = sin^{-1}\left(\frac{T}{m_1 g}\right) = sin^{-1}\left(\frac{2.45}{1\times 9.8}\right) = 14.5^0$

$F = 2.45\,N$

13. A solid sphere of mass 0.1 kg falls through a resistive medium in which resistive force is proportional to the velocity of the sphere. It attains a terminal velocity of 2 ms^{-1}. Calculate the drag coefficient and time constant.

Given data : $m = 0.1\,kg,\ (F \propto v),\ v_t = 2\,ms^{-1},\ C = \ ?\quad \tau = \ ?$

Formula : $v_t = \frac{mg}{C}\qquad \tau = m/c$

Solution : $C = \frac{mg}{v_t} = \frac{0.1\times 9.8}{2} = 0.49$ **C = 0.49**

$\tau = \frac{m}{C} = \frac{0.1}{0.49} = 0.204\,s$ **τ = 0.204 s**

14. Estimate the drag force on an automobile cruising at 100 kmph. Assume that the drag coefficient C_D is 0.45 and that the car's cross-sectional area is $4\,\mathrm{m}^2$. Take air to have a density of $1.25\,\mathrm{kg\,m^{-3}}$. Assume resistance varies quadratically with velocity.

Given data : $v = 100\,kmph = \frac{100\times 1000}{60\times 60} = 27.77\,ms^{-1},\ C_D = 0.45,$

$A = 4m^2,\ \rho = 1.25\,kgm^{-3}$

Formula : $F_D = \frac{1}{2}c\,\rho A v^2$

Solution : $F_D = \frac{1}{2}\times 0.45\times 1.25\times 4\times 27.77\times 27.77$ $F_D = 867.56\,N$

15. A small 150 g pebble is 3.0 km deep in ocean and is falling with a constant terminal velocity of 30 ms^{-1}. What force does water exert on the falling pebble? Find the drag coefficient.

Given data : $m = 150\,g = 0.15\,kg,\ h = 3\,km,\ v_t = 30ms^{-1},\ F = \ ?\ \ c = \ ?$

Formula : $F_D = W = mg$ and $v_t = \frac{mg}{c}$

Solution : $F_D = 0.15\times 9.8 = 1.47\,N$

$c = \frac{mg}{v_t} = \frac{1.47}{30} = 0.049$

16. An object is dropped in a viscous medium in which resistance varies directly as velocity of the object. How long does it take for the object to reach one – fourth of its terminal velocity? Given : Mass of the object = 100 g and drag coefficient = 0.2 Nsm^{-1}. Also, calculate the terminal velocity.

Given data : $m = 100\,g = 0.1\,kg,\ c = 0.2,\ \ v = \frac{1}{4}v_t,\ t = \ ?$

Formula : $v = v_t\left(1 - e^{-\frac{c}{m}t}\right)$ and $v_t = \frac{mg}{c}$

Solution : $v = v_t\left(1 - e^{-\frac{c}{m}t}\right)$ or $\frac{1}{4}v_t = v_t\left(1 - e^{-\frac{c}{m}t}\right)$ or $1 - e^{-\frac{c}{m}t} = \frac{1}{4} = 0.25$

or $e^{-\frac{c}{m}t} = 0.75$ or $e^{\frac{c}{m}t} = \frac{1}{0.75} = 1.34$ or $\frac{c}{m}t = log_e 1.34 = 2.303\,log_{10}1.34 = 0.29$

or $t = 0.29\times\frac{0.1}{0.2} = 0.145\,s$ $v_t = \frac{0.1\times 9.8}{0.2} = 4.9\,ms^{-1}$

17. A raindrop with radius 1.2mm falls from a cloud that is at a height of 1.2 km above ground. The drag coefficient is 0.6. Calculate the terminal speed of the drop if density of water is 1000 kgm^{-3} and density of air is 1.23 kg m^{-3}.

Given data : $r = 1.2mm = 1.2 \times 10^{-3}m, \quad h = 1.2km,$

$C = 0.6, \;\rho' = 1000kgm^{-3} \quad v_t = \;? \quad \rho = 1.23\;kgm^{-3}$

Formula : $v_t = \sqrt{\frac{2mg}{C\rho A}}$ and $A = \pi r^2$ Also $m = \rho' \times V = \rho' \times \frac{4}{3}\pi r^3$

Solution : $A = 3.14 \times 1.2^2 \times 10^{-6} = 18.92 \times 10^{-6}m^2$

$m = 1000 \times \frac{4}{3} \times 3.14 \times 1.2^3 \times 10^{-9} = 7.23 \times 10^{-6}kg$

$v_t = \sqrt{\frac{2\times 7.23\times 10^{-6}\times 9.8}{0.6\times 1.23\times 18.92\times 10^{-6}}} = 3.18\;ms^{-1}$

18. A sphere of mass 3 g moving vertically downwards in a resistive medium has the terminal velocity of 0.05 ms^{-1} and the drag coefficient is 0.6. Calculate the time constant and the time taken by the sphere to reach 80%of its terminal speed. (Assume the resistive force is proportional to its velocity)

Given data : $m = 3\;g = 3 \times 10^{-3}kg, \;v_t = 0.05\;ms^{-1}, \;c = 0.6, \;\tau = \;? \quad v = \frac{80}{100}v_t$

Formula : $v = v_t\left(1 - e^{-\frac{t}{\tau}}\right)$ and $\tau = \frac{m}{c}$

Solution : Time constant $\tau = \frac{m}{c} = \frac{3\times 10^{-3}}{0.6} = 0.5 \times 10^{-2}s$

$v = v_t\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{80}{100}v_t\left(1 - e^{-\frac{t}{\tau}}\right)$ or $0.8 = 1 - e^{-\frac{t}{\tau}}$ *or* $e^{-\frac{t}{\tau}} = 0.2$

or $e^{\frac{t}{\tau}} = \frac{1}{0.2} = 5 \qquad \frac{t}{\tau} = 2.303\;log_{10}5 \qquad t = \tau \times 2.303\;log_{10}5 \qquad t = 0.805 \times 10^{-2}s$

19. A parachutist of mass 90 kg finds that his terminal speed is 6 ms^{-1}. If the effective area of the parachute is 30 m^2, find the drag coefficient. Assume the density of air to be 1.25 kgm^3.

Given data :$m = 90\;kg, \;v_t = 6\;ms^{-1}, \;A = 30m^2, \;\rho = 1.25kgm^{-3}\;, \;c = \;?$

Formula : $c = \frac{2mg}{\rho A v_t^2}$

Solution : $c = \frac{2\times 90\times 9.8}{1.25\times 30\times 6\times 6} = 1.306$

20. Determine the drag coefficient of a 75 kg skydiver with a projected area of 0.33 m^2 and a terminal velocity of 60 ms^{-1}. By how much would the skydiver need to reduce his projected area so as to double his terminal velocity?

Given data : $m = 75\;kg, \quad A = 0.33\;m^2, \;v_t = 60\;ms^{-1}c = \;?$

Formula : $v_t = \sqrt{\frac{mg}{c}}$ or $c = \frac{mg}{v_t^2} \qquad v_t \propto \frac{1}{\sqrt{A}}$ *or* $A \propto \frac{1}{v_t^2}$

Solution : $c = \frac{mg}{v_t^2} = \frac{75\times 9.8}{60\times 60} = 0.204$ As $v_t \propto \frac{1}{\sqrt{A}}$ *If* v_t is doubled, the area is reduced to 1/4th.

Non-Inertial Frame of reference

- Reference frames in which Newton's laws are not valid.
- They are accelerating frames.
- Such an accelerating frame of reference is called a non-inertial frame because the law of inertia does not hold in it.
- Its velocity is *not* constant. So, it is either *changing* its *speed* by speeding up or slowing down, or it is *changing* its *direction* by traveling in a curved path, or it is both changing its speed and changing its direction.
- Here zero force doesn't correspond to zero acceleration
- In a non-inertial frame of reference fictitious forces arise.

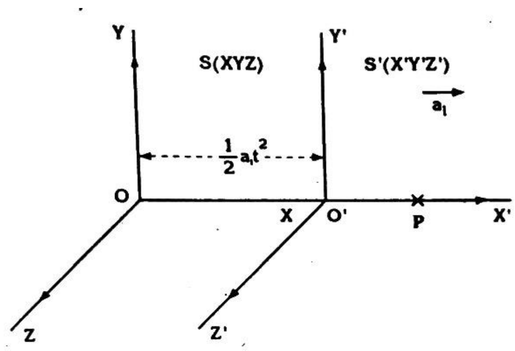
Explanation : A person in the car is pushed forward when brakes are abruptly applied. In reality, no force is pushing the person forward.

The car, which is slowing down, is an accelerating, or non-inertial frame of reference. The law of inertia no longer holds in this non-inertial frame.

With respect to the ground, which is at rest, no force is pushing the person forward when the brakes are applied. The ground is an inertial frame. Relative to this frame, when the brakes are applied, the person continues with forward motion, according to Newton's first law of motion.

From the point of view of the person in the car, he is pushed spontaneously forward. Actually, there is no force acting on him. This imaginary force acting on the person is called fictitious or pseudo force or the frame dependent force.

Two frames of reference in relative motion with uniform acceleration (Accelerated frames)



S (inertial frame) and S' (non inertial frame) are the two frames with S at rest and S' moving with uniform acceleration a_i.

Let a particle P of mass m be moving along + X direction with an acceleration a as measured by frame S. Since the frame inertial Newton second law is verified. The observer O in S measures force on particle P as $F = ma$

Initially both frames are at rest and coincide.

Let frame S' start from origin. After some time t, the position of particle P is $OP = OO' + O'P$ **i.e.** $\boldsymbol{x = \frac{1}{2}a_it^2 + x'}$ **....(1)**

or $x' = x - \frac{1}{2}a_it^2$ ....(2)

Differentiating eqn. (2), $\frac{dx'}{dt} = \frac{dx}{dt} - a_i t$

Differentiating again, $\frac{d^2x'}{dt^2} = \frac{d^2x}{dt^2} - a_i$ or $a' = a - a_i$

or $ma' = ma - ma_i$ ....(3)

The real force acting on the particle is $F = ma$

The frame S' measures the force as $F' = ma'$

Hence eqn. (3) is $\boldsymbol{F' = F - F_i'}$ **or** $\boldsymbol{F' = F + F_i'}$ where $F_i' = -ma_i$

The force F_i' is the additional force that acts in an accelerated force which is not real called **pseudo or fictitious force**.

This is fictitious in the sense that, their origin cannot traced but it produces same effect as that of any real force.

Example : Let a person be in a lift which going down with acceleration a_i (frame S') in the y direction w.r.t to a fixed frame S.

The relation is $\frac{d^2y'}{dt^2} = \frac{d^2y}{dt^2} - a_i$ or $g' = g - a_i$

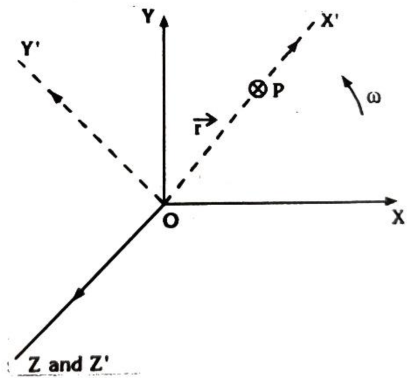
or $mg' = mg - ma_i$ or $W' = W - ma_i$

where, $W' = mg'$ *and* $W = mg$ (as measured by inertial frame S)

Thus when the lift is accelerating downwards, the person feels loss weight or lighter. Also when the lift is moving upwards, he feels heavier as $W' = W + ma_i$ Thus the person in the accelerated frame experiences this fictitious force or pseudo force.

Rotating frames

Expressions for Coriolis force and Centrifugal force



Consider the rotational motion of a particle P. Let its motion be considered as seen by a rotating frame (non inertial) and a inertial frame.

Let S (XYZ) be an inertial frame and S'(X'Y'Z') be a rotating frame with both having a common origin. Z and Z' axes always coincide.

Also S' rotate about Z' with an angular velocity ω.

Let a particle P of mass m rotate in the XY plane about the Z axis. $\overrightarrow{OP} = \vec{r}$

P has an angular acceleration of $\omega^2 r$ along $\overrightarrow{PO}$.

The position vector of P in S frame $\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$ ...(1)

The position vector of P in S' frame $\vec{r'} = x'\,\hat{\imath'} + y'\,\hat{\jmath'} + z'\,\hat{k'}$ ...(2)

As both S and S' have same origin, $\vec{r} = \vec{r'}$ or $\vec{r}]_{Fixed} = \vec{r}]_{Rot}$

Thus $\vec{r} = x'\,\hat{\imath'} + y'\,\hat{\jmath'} + z'\,\hat{k'}$ ...(3)

The time derivative of $\vec{r}$ and $\vec{r'}$ must also be equal as they are one and the same physical vector $\frac{d}{dt}(\vec{r})_{Fixed} = \frac{d}{dt}(\vec{r})_{Rot}$

$\left(\frac{d\vec{r}}{dt}\right)_{Fixed} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k}$ & $\left(\frac{d\vec{r}}{dt}\right)_{Rot} = \frac{dx'}{dt}\hat{\imath'} + \frac{dy'}{dt}\hat{\jmath'} + \frac{dz'}{dt}\hat{k'}$

Differentiating eqn. (3),

$\left(\frac{d\vec{r}}{dt}\right)_{Fixed} = \frac{dx'}{dt}\hat{\imath'} + \frac{dy'}{dt}\hat{\jmath'} + \frac{dz'}{dt}\hat{k'} + x'\frac{d\hat{\imath'}}{dt} + y'\frac{d\hat{\jmath'}}{dt} + z'\frac{d\hat{k'}}{dt}$ ...(4)

With respect to observer in S, S' is rotating and unit vectors are also changing. The vectors are rotating with angular velocity ω relative to S. Hence, $\frac{d\hat{\imath'}}{dt} = \omega \times \hat{\imath'}$,

$\frac{d\hat{\jmath'}}{dt} = \omega \times \hat{\jmath'}$, $\frac{d\hat{k'}}{dt} = \omega \times \hat{k'}$ ....(6)

(Also $\left(\frac{d\vec{r}}{dt}\right)_{Fixed} = \left(\frac{d\vec{r}}{dt}\right)_{S}$ & $\left(\frac{d\vec{r}}{dt}\right)_{Rot} = \left(\frac{d\vec{r}}{dt}\right)_{S'}$) Thus eqn. (4) is

$\left(\frac{d\vec{r}}{dt}\right)_{S} = \left[\frac{dx'}{dt}\hat{\imath'} + \frac{dy'}{dt}\hat{\jmath'} + \frac{dz'}{dt}\hat{k'}\right] + \left[(\omega \times \hat{\imath'})x' + (\omega \times \hat{\jmath'})y' + (\omega \times \hat{k'})z'\right]$ ..(5)

or $\left(\frac{d\vec{r}}{dt}\right)_{S} = \left[\frac{dx'}{dt}\hat{\imath'} + \frac{dy'}{dt}\hat{\jmath'} + \frac{dz'}{dt}\hat{k'}\right] + \omega \times \left[x'\hat{\imath'} + y'\hat{\jmath'} + z'\hat{k'}\right]$ ...(6)

The first term in the RHS is the rate of change of position vector as measured in S' frame. Thus $\left[\frac{dx'}{dt}\hat{\imath'} + \frac{dy'}{dt}\hat{\jmath'} + \frac{dz'}{dt}\hat{k'}\right] = \left(\frac{d\vec{r}}{dt}\right)_{S'} = \vec{v}'$

As $\vec{r} = x'\hat{\imath'} + y'\hat{\jmath'} + z'\hat{k'}$,

Eqn. (6) is $\left(\frac{d\vec{r}}{dt}\right)_{S} = \left(\frac{d\vec{r}}{dt}\right)_{S'} + \omega \times \vec{r}$ ...(7)

Here $\left(\frac{d\vec{r}}{dt}\right)_{S} = \vec{v}$. *Thus* $\vec{v} = \vec{v}' + (\omega \times \vec{r})$ ...(8) or $\vec{v} = \vec{v}' + \vec{v}_0$

$\boldsymbol{\vec{v} = \vec{v}' + (\omega \times \vec{r})}$ *where $\vec{v}$ is the velocity of the particle P with respect to inertial or fixed frame S and $\vec{v}'$ is the velocity with respect to non inertial or rotating frame.*

Also $\boldsymbol{\vec{v} = \vec{v}' + \vec{v}_0}$ where $\boldsymbol{\vec{v}_0 = (\omega \times \vec{r})}$

Thus, the velocity $\vec{v}$ of the particle in the inertial frame is the **vector sum** of its velocity in the rotating frame $\vec{v}'$ and the linear velocity due to rotation ($\vec{v}_0$), the value of which at any instant depends on the value of $\vec{r}$ at that instant.

Operator equation – The relation $\boldsymbol{\left(\frac{d\vec{r}}{dt}\right)_{S} = \left(\frac{d\vec{r}}{dt}\right)_{S'} + \omega \times \vec{r}}$ may be conveniently expressed by the operator identity $\boldsymbol{\left(\frac{d}{dt}\right)_{S} = \left(\frac{d}{dt}\right)_{S'} + \vec{\omega} \times}$ ......(9)

Applying the operator equation to the velocity vector in place of displacement vector,

$\boldsymbol{\left(\frac{d\vec{v}}{dt}\right)_{S} = \left(\frac{d\vec{v}}{dt}\right)_{S'} + \omega \times \vec{v}}$ ...(10)

Substituting for $\boldsymbol{\vec{v}}$ from eqn. (8) i. e. $\boldsymbol{\vec{v} = \vec{v}' + (\omega \times \vec{r})}$ in (10)

$$\boldsymbol{\left(\frac{d\vec{v}}{dt}\right)_{S} = \frac{d}{dt}(\vec{v}' + \omega \times \vec{r})_{S'} + \omega \times (\vec{v}' + \omega \times \vec{r})_{S'}}$$

$$= \frac{d\vec{v}'}{dt} + \left(\frac{d\omega}{dt} \times \vec{r}\right)_{S'} + \omega \times \left(\frac{d\vec{r}}{dt}\right)_{S'} + \omega \times \vec{v}' + \omega \times (\omega \times \vec{r})$$

$$= \frac{d\vec{v}'}{dt} + \left(\frac{d\omega}{dt} \times \vec{r}\right)_{S'} + \omega \times \vec{v}' + \omega \times \vec{v}' + \omega \times (\omega \times \vec{r})$$

$$\left(\frac{d\vec{v}}{dt}\right)_S = \frac{d\vec{v}'}{dt} + 2(\omega \times \vec{v}') + \omega \times (\omega \times \vec{r}) + \left(\frac{d\omega}{dt} \times \vec{r}\right)_{S'}$$

Or $\boldsymbol{\vec{a} = \overrightarrow{a'} + 2(\omega \times \vec{v}') + \omega \times (\omega \times \vec{r}) + \left(\frac{d\omega}{dt} \times \vec{r}\right)_{S'}}$ ....(11)

Here $\boldsymbol{\overrightarrow{a'}}$ is the acceleration of the particle as measured from rotating frame S and $\boldsymbol{\vec{a}}$ is the acceleration as measured from fixed frame S'

In the eqn. (11) the term $\boldsymbol{2(\vec{\omega} \times \vec{v}')}$ is called the **Coriolis acceleration** and $\boldsymbol{\omega \times (\omega \times \vec{r})}$ is called the **Centripetal acceleration**.

For a uniform angular velocity $\boldsymbol{\frac{d\omega}{dt} = 0}$.

Thus $\boldsymbol{\vec{a} = \overrightarrow{a'} + 2(\omega \times \vec{v}') + \omega \times (\omega \times \vec{r})}$

or $\boldsymbol{\overrightarrow{a'} = \vec{a} - 2(\overrightarrow{\omega} \times \vec{v}') - \omega \times (\omega \times \vec{r})}$ ...(12)

The term $\boldsymbol{-\omega \times (\omega \times \vec{r})}$ is called **Centrifugal acceleration**.

As $\overrightarrow{a'} = \vec{a} - 2(\overrightarrow{\omega} \times \vec{v}') - \omega \times (\omega \times \vec{r})$

And if m is the mass of the particle,

$m\overrightarrow{a'} = m\vec{a} - 2m(\overrightarrow{\omega} \times \vec{v}') - m\omega \times (\omega \times \vec{r})$

or $\overrightarrow{F'} = \vec{F} - 2m(\overrightarrow{\omega} \times \vec{v}') - m\omega \times (\omega \times \vec{r})$ ......(13)

In the above equation $\boldsymbol{-2m(\overrightarrow{\omega} \times \vec{v}')}$ **is called the Coriolis force** and $\boldsymbol{-m\omega \times (\omega \times \vec{r})}$ **is called the centrifugal force.** These forces are frame dependent forces arising in non inertial frame and are pseudo forces or fictitious forces.

The equation $\overrightarrow{F'} = \vec{F} - 2m(\overrightarrow{\omega} \times \vec{v}') - m\omega \times (\omega \times \vec{r})$

can be expressed as $\overrightarrow{F'} = \vec{F} + \vec{F}_0$

where $\overrightarrow{F'} = m\overrightarrow{a'}$ is the effective force on the particle m in the rotating frame S' .

$\vec{F} = m\vec{a}$ is the true force on the particle in the inertial frame S.

and $\vec{F}_0 = -2m(\overrightarrow{\omega} \times \vec{v}') - m\omega \times (\omega \times \vec{r})$ is the fictitious force in the non inertial frame S'. Thus $Fictitious\ force = Coriolis\ force + Centrifugal\ force$

Coriolis force

The Coriolis force given by $\boldsymbol{F_C = -2m(\overrightarrow{\omega} \times \vec{v}')}$ is defined as the fictitious force that acts on a particle when it is in motion relative to a rotating frame of reference.

$\boldsymbol{F_C}$ is proportional to the angular velocity $\boldsymbol{\overrightarrow{\omega}}$ of the rotating frame and also to the velocity $\boldsymbol{\vec{v}'}$ of the particle relative to it.

The Coriolis force is zero if (1) $\boldsymbol{\overrightarrow{\omega} = 0}$, i.e. the reference frame is non rotating and (2) $\boldsymbol{\vec{v}' = 0}$, i.e. if the particle is at rest relative to the rotating frame.

Also the cross product of $\overrightarrow{\omega}\ and\ \vec{v}'$ indicates that the direction of this force is perpendicular to both angular velocity of rotating frame and velocity of particle with respect to rotating frame.

In the equation $\boldsymbol{F_C} = -\boldsymbol{2m}(\overrightarrow{\boldsymbol{\omega}} \times \vec{\boldsymbol{v}}')$, the negative sign indicates that the direction of Coriolis force is opposite to that given by right handed screw rule.

This force can be represented as

$\boldsymbol{F_C} = -\boldsymbol{2m}(\overrightarrow{\boldsymbol{\omega}} \times \vec{\boldsymbol{v}}') = -\boldsymbol{2m}\overrightarrow{\boldsymbol{\omega}}\vec{\boldsymbol{v}}' \,\boldsymbol{sin\theta}\ \hat{\boldsymbol{n}}$

Special cases : 1) If $\boldsymbol{\theta = 0}$, Coriolis force is zero. Thus when the angular velocity vector of the rotating frame is parallel to the particle velocity relative to the rotating frame, the Coriolis force vanishes.

2) If $\boldsymbol{\theta} = \frac{\pi}{2}, then\ sin\theta = 1$. The Coriolis force is $\boldsymbol{F_C} = \boldsymbol{2m}\overrightarrow{\boldsymbol{\omega}}\vec{\boldsymbol{v}}'$ which is maximum. The Coriolis force is maximum when $\overrightarrow{\boldsymbol{\omega}}$ $\boldsymbol{and}\ \vec{\boldsymbol{v}}'$ are at right angles to each other.

Centrifugal force

A centripetal force $\boldsymbol{F_c} = \boldsymbol{m\omega^2 r}$ acts along $\overrightarrow{\boldsymbol{PO}}$ on particle P **according to frame S** and no other force acts on it. ($F = \frac{mv^2}{r}$, Also $v = r\omega$)

According to observer in S' frame, the particle P is at rest as S' is rotating with the same angular velocity as that of P.

An additional force $\boldsymbol{m\omega^2 r}$ acts on the particle along $\overrightarrow{\boldsymbol{OP}}$ (outwards) in addition to centripetal $\boldsymbol{F_c} = \boldsymbol{m\omega^2 r}$ along $\overrightarrow{\boldsymbol{PO}}$ (inwards). Thus P is at rest.

This additional force is called **frame dependent force** or **centrifugal force**. (It is a pseudo force or a fictitious force)

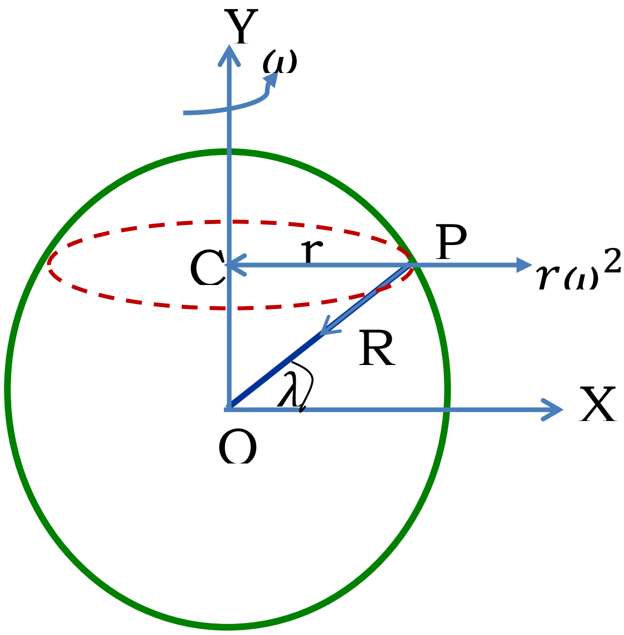
The term $\boldsymbol{F_{CF}} = -\boldsymbol{m\omega} \times (\boldsymbol{\omega} \times \vec{\boldsymbol{r}})$ arises as a result of the rotation of co-ordinate axes and the centrifugal force is defined as the fictitious force that acts on a **particle at rest** relative to a rotating frame of reference.

It is numerically equal to the centripetal force but acts in the opposite direction i.e. outwards and away from the rotational axis. Its magnitude is given by $\boldsymbol{F_{CF}} = \boldsymbol{m\omega^2 r}$

Note : 1. Centrifugal force exists only when viewed from the rotating frame. It has no existence in a inertial frame.

2. Coriolis force always acts at right angles to the direction of motion of the particle, and does no work. It only changes the direction of motion of the particle without changing the speed.

Effect of Centrifugal force due to Earth's rotation on g



Earth rotates with a constant angular velocity $\vec{\omega}$ from west to east and the frame fixed to it is a rotating frame.

The particle under consideration is assumed to be at rest so that there is no Coriolis force. Only the effect of centrifugal acceleration is considered on $\vec{g}$, the acceleration due to gravity.

Let $\vec{\omega}$ be along Y direction. The value of $\vec{g}$ at a point P on earth's surface at latitude λ will be along PO with O being the centre of earth. R is radius of earth.

The component of $\vec{g}$ are $-gcos\lambda\hat{\imath}$ along X-axis and $-gsin\lambda\hat{\jmath}$ along Y-axis. Therefore $\vec{g} = -g(cos\lambda\hat{\imath} + sin\lambda\hat{\jmath})$ Also $\vec{\omega} = \omega\hat{\jmath}$

A perpendicular PC is drawn from P to OY. Then $CP = \vec{r}$, the radius of the circle in which a particle at P rotates. Therefore $\overrightarrow{CP} = \vec{r} = \vec{R}cos\lambda = Rcos\lambda\hat{\imath}$

The observed g – value at P, taking centrifugal force into account , is

$\overrightarrow{g'} = \vec{g} - \vec{\omega} \times (\vec{\omega} \times \vec{r})$ (since $\overrightarrow{a'} = \vec{a} - \omega \times (\omega \times \vec{r})$)

As $\vec{g} = -g(cos\lambda\hat{\imath} + sin\lambda\hat{\jmath})$, $\vec{\omega} = \omega\hat{\jmath}$ and $\vec{r} = Rcos\lambda\hat{\imath}$

The eqn. $\overrightarrow{g'} = \vec{g} - \vec{\omega} \times (\vec{\omega} \times \vec{r})$ will be

$$\overrightarrow{g'} = -g(cos\lambda\hat{\imath} + sin\lambda\hat{\jmath}) - \omega\hat{\jmath} \times (\omega\hat{\jmath} \times Rcos\lambda\hat{\imath})$$

$$= -\{(gcos\lambda - \omega^2 Rcos\lambda)\hat{\imath} + gsin\lambda\hat{\jmath}\}$$

$$\overrightarrow{g'}^2 = \overrightarrow{g'}.\overrightarrow{g'} = \{(gcos\lambda - \omega^2 Rcos\lambda)^2 + g^2 sin^2\lambda\}$$

$$= g^2cos^2\lambda\left(1 - \frac{\omega^2 R}{g}\right)^2 + g^2 sin^2\lambda$$

$$= g^2cos^2\lambda\left(1 - \frac{2\omega^2 R}{g}\right) + g^2 sin^2\lambda \qquad (\because \frac{\omega^2 R}{g} \ll 1)$$

$$\overrightarrow{g'}^2 = g^2cos^2\lambda\left(1 - \frac{2\omega^2 R}{g}\right) + g^2 sin^2\lambda$$

$g' = g\left[cos^2\lambda\left(1 - \frac{2\omega^2 R}{g}\right) + sin^2\lambda\right]^{1/2}$ (by taking square root on both sides)

or $g' = g\left[1 - \frac{2\omega^2 R}{g}cos^2\lambda\right]^{1/2}$ $\boldsymbol{g' = g\left[1 - \frac{\omega^2 R}{g}cos^2\lambda\right]}$

$\boldsymbol{g' = g - \omega^2 Rcos^2\lambda}$ in magnitude

Thus due to the rotation of the earth, the value of g at latitude λ decreases by a value $\omega^2 Rcos^2\lambda$

$\boldsymbol{g' = g - \omega^2 R cos^2 \lambda}$ At the equator, where $\lambda = 0$, $cos^2\lambda = 1$ the decrease in the g – value is maximum and the decrease in g is

$\omega^2 R = \left(\frac{2\pi}{24\times60\times60}\right)^2 \times 6400 \times 10^3 = 0.0337 ms^{-2}$

At the poles, where $\lambda = 90^0$, $cos^2\lambda = 0$, there is no decrease in the value of g due to rotation of earth at the poles.

At the latitude of $\lambda = 45^0$, the decrease in the value of g is $\omega^2 R cos^2 45 =$

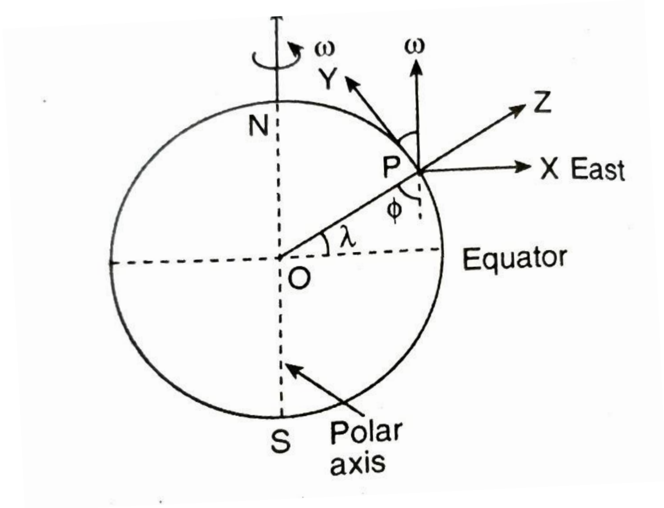
$\omega^2 R\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}\omega^2 R = \frac{1}{2}\times 0.0337 = 0.0168\, ms^{-2}$.

The effect of centrifugal force due to the rotation of earth is minimum (zero) at the poles and maximum at the equator.

Effect of Coriolis force on a moving particle on surface of Earth

The Coriolis force acting on a particle of mass m moving with velocity $\vec{v}$ in a frame rotating with angular velocity $\vec{\omega}$ is

$$F_C = -2m(\overrightarrow{\omega} \times \vec{v}')$$



Let P be a point on the surface of earth at a latitude λ .

The cartesian coordinate system XYZ is fixed to earth at point P and the Z – axis is taken vertically upwards at P (radially outwards from the centre of earth).

The xy – plane is horizontal containing P, the + X axis pointing east and +Y axis pointing north and earth rotates from west to east.

The angular velocity vector $\vec{\omega}$ is in yz – plane directed parallel to the polar axis NS about which earth rotates. Thus $\vec{\omega}$ has no component along east –west i.e. along X – axis.

$\vec{\omega} = \omega_x\hat{\imath} + \omega_y\hat{\jmath} + \omega_z\hat{k}$

$= \omega_y\hat{\jmath} + \omega_z\hat{k}$ as $\omega_x = 0$

$\vec{\omega} = \omega cos\lambda\hat{\jmath} + \omega sin\lambda\hat{k}$

Let a particle be projected from P horizontally with a velocity $\vec{v}$.

Then $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$

$= \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$

$= \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$ as $v_z = \dot{z} = 0$

The Coriolis force on the moving particle is $\overrightarrow{F_C} = -2m(\overrightarrow{\omega} \times \vec{v})$

$$\overrightarrow{F_C} = -2m\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & \omega_y & \omega_z \\ v_x & v_y & 0 \end{vmatrix} = -2m\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & \omega cos\lambda & \omega sin\lambda \\ \dot{x} & \dot{y} & 0 \end{vmatrix}$$

$\overrightarrow{F_C} = -2m\left(-\dot{y}\omega sin\lambda\hat{\imath} + \dot{x}\omega sin\lambda\hat{\jmath} - \dot{x}\omega cos\lambda\hat{k}\right)$

$\overrightarrow{F_C} = -2m\omega\left(-\dot{y}sin\lambda\hat{\imath} + \dot{x}sin\lambda\hat{\jmath} - \dot{x}cos\lambda\hat{k}\right)$

The magnitude of the Coriolis force is given by

$\overrightarrow{F_C} = 2m\omega(\dot{y}^2 sin^2\lambda + \dot{x}^2 sin^2\lambda + \dot{x}^2 cos^2\lambda)^{1/2}$

$\overrightarrow{F_C} = 2m\omega\left(v_y^2 sin^2\lambda + v_x^2\right)^{1/2}$ in terms of latitude λ

The horizontal component of Coriolis force is given by

$\left(\overrightarrow{F_C}\right)_H = -2m\omega(-\dot{y}sin\lambda\hat{\imath} + \dot{x}sin\lambda\hat{\jmath})$

$= -2m\omega sin\lambda\left(-\dot{y}\hat{\imath} + \dot{x}\hat{\jmath}\right)$

The magnitude of horizontal component of Coriolis force is

$\left|\left(\overrightarrow{F_C}\right)_H\right| = 2m\omega sin\lambda(\dot{y}^2 + \dot{x}^2)^{1/2}$

$= 2m\omega sin\lambda\left(v_y^2 + v_x^2\right)^{1/2}$ Also $\left(v_y^2 + v_x^2\right)^{1/2} = v$

$= 2m\omega v sin\lambda$ .... In terms of latitude λ

$= 2m\omega v cos\phi$ .... In terms of co-latitude ϕ

The horizontal component of Coriolis force has maximum value at the poles ($\lambda = 90^0$) and zero at the equator ($\lambda = 0$)

The vertical component of Coriolis force is given by

$\left(\overrightarrow{F_C}\right)_V = -2m\omega\left(-\dot{x}cos\lambda\hat{k}\right)$

$= +2m\omega\, \dot{x}cos\lambda\hat{k}$

The magnitude of vertical component of Coriolis force is

$\left|\left(\overrightarrow{F_C}\right)_V\right| = 2m\omega cos\lambda(\dot{x}^2)^{1/2} = 2m\omega cos\lambda(v_x^2)^{1/2}$

$= 2m\omega\, v_x cos\lambda$ .... In terms of latitude λ

$= 2m\omega\, v_x sin\phi$ .... In terms of co-latitude ϕ

The vertical component ($= +2m\omega\, \dot{x}cos\lambda\hat{k}$) of Coriolis force acts in the direction of +Z axis which means vertically upwards. At the equator if it has an initial velocity, it will appear to be lifted upwards.

Direction of Coriolis force in the northern and southern hemisphere

The direction of angular velocity $\vec{\omega}$ of rotation of earth is always along the axis of earth i.e. from south to north. In the northern hemisphere, if a body is moving towards north, i.e. along the +Y axis, applying the rule of vector product of two vectors, the Coriolis force will act towards east i.e. along +X axis.

If the body is moving towards south, i.e. along -Y axis, the Coriolis force will act along west, i.e. along –X axis. Thus in northern hemisphere, a moving body turns towards right and in the southern hemisphere a moving body turns towards left due to the effect of Coriolis force.

Geophysical effects of Coriolis force

1. Direction of Trade winds : The air that gets heated at the equator due to sun-rays, rises upwards and cooler air from poles (north and south) move towards the equator creating pressure gradients.

Due to Coriolis force, the wind does not move straight instead it deflected towards its right i.e. in west direction in the northern hemisphere and towards east in the southern hemisphere.

2. Cyclones : Low pressure regions created in the atmosphere due to differential heating of air is called cyclone. In the northern hemisphere, the iar from all directions rushes towards low pressure region. This air is deflected towards right due to Coriolis force. This causes anticlockwise rotation or whirling of wind round the low pressure area.

This rotational motion goes on till the thrust due to pressure gradient is balanced by that due to Coriolis force.

In the southern hemisphere, the cyclonic direction is in the clockwise direction.

At equator, no cyclone occurs as the horizontal component of Coriolis force
$2m\omega v sin\lambda = 0$ as the latitude λ at the equator is zero.

3. Erosion of right bank of rivers : The water of rivers with their courses from north to south or from south to north will experience Coriolis force towards the right of flow direction in the northern hemisphere and to the left of the flow direction in the southern hemisphere due to the rotation of earth. Thus the right bank of the river is eroded more rapidly compared to the left bank.

Foucault pendulum

The Foucault pendulum is a simple device named after French physicist Léon Foucault and is an experiment to demonstrate the Earth's rotation.

Foucault also observed that the small effect of Coriolis force could be greatly amplified by using a pendulum.

He noticed that the rightward Coriolis deflection on one swing of the pendulum cannot be nullified in the return swing.

Foucault pendulum consists of relatively large mass (around 28kg iron sphere) suspended from a long steel wire (around 67 m) mounted so that its perpendicular plane of swing is not confined to a particular direction and can rotates in relation to the Earth's surface.

While a Foucault pendulum swings back and forth in a plane, the Earth rotates beneath it, so that relative motion exists between them.

At the North pole, latitude 90° N, the relative motion as viewed from above in the plane of the pendulum's suspension is a counterclockwise rotation of the Earth once approximately every 24 hours Correspondingly, the plane of the pendulum as viewed from above appears to rotate in a clockwise direction once a day.

A Foucault pendulum always rotates clockwise in the Northern Hemisphere with a rate that becomes slower as the pendulum's location approaches the equator. Foucault's original pendulums at Paris rotated clockwise at a rate of more than 11° per hour, or with a period of about 32 hours per complete rotation.

The rate of rotation depends on the latitude. At the Equator, 0° latitude, a Foucault pendulum does not rotate.

In the Southern Hemisphere, rotation is counterclockwise.

The period of rotation of the pendulum is $T = \frac{2\pi}{\omega\, sin\lambda}$

At the pole $\lambda = 90^0$, T = 24 hours.. Thus the plane of oscillation of the pendulum at the poles makes a complete revolution in 24 hours.

At the equator $\lambda = 0^0$, $T = \infty$, $i.e.$ There is no rotation of the plane of oscillation of the pendulum at the equator. At any other latitude, the period $T > 24\, hours$.

By measuring T, Foucault determined the period of rotation of the earth and thus provided the terrestrial demonstration of earth's axial rotation.

1. Calculate the fictitious force and the total force acting on a mass of 3 kg in a frame of reference moving (i) vertically downwards and (ii) vertically upwards with an acceleration of $4ms^{-2}$. The acceleration due to gravity is $9.8ms^{-2}$.

Taking the earth to be inertial and upward direction as positive, the weight of the body $F = mg = 3 \times (-9.8) = -29.4\, N$.

(i) For downward motion, the fictitious force $F_0 = -ma_0 = -3 \times (-4) = 12\, N\, (upwards)$

Thus, total force $F' = F + F_0 = -29.4 + 12 = -17.4\, N\, (downwards)$

(ii) For upward motion, the fictitious force $F_0 = -ma_0 = -3 \times (+4) = -12\, N\, (upwards)$

Thus, total force $F' = F + F_0 = -29.4 - 12 = -41.4\, N\, (downwards)$

2. A rocket of mass 5000 kg is fired vertically upwards from a place at the equator with a velocity of $1200ms^{-1}$. If the angular velocity of the earth is $7.3 \times 10^{-5} rads^{-1}$, calculate the Coriolis force acting on it.

Coriolis force $F_C = 2m(\overrightarrow{\omega} \times \vec{v}') = 2m\, \omega\, v\, sin\theta = 2m\, \omega\, v$ As $sin\theta = 90^0$

$F_C = 2 \times 5000 \times 7.3 \times 10^{-5} \times 1200 = 876\, N$

Newton's laws of motion and their applications

Multiple choice questions

1. The inertia of a moving object depends on:
i. Mass of the object
ii. Momentum of the object
iii. Speed of the object
iv. Shape of the object
Choose the correct option:
(a) (i) and (ii)
(b) only (i)
(c) only (ii)
(d) (iii) and (iv)

2. The seat belts are provided in the cars so that if the car stops suddenly due to an emergency braking, the persons sitting on the front seats are not thrown forward violently and saved from getting injured. The law due to which a person falls in forward direction is
(a) Newton's first law of motion
(b) Newton's second law of motion
(c) Newton's third law of motion
(d) Newton's law of gravitation

3. Which of the following situations involves the Newton's second law of motion?
(a) A force can stop a lighter vehicle as well as a heavier vehicle which are moving
(b) A force exerted by a lighter vehicle on collision with a heavier vehicle results in both the (vehicles coming to a standstill
(c) A force can accelerate a lighter vehicle more easily than a heavier vehicle which are moving
(d) A force exerted by the escaping air from a balloon in the downward direction makes the balloon to go upwards

4. A passenger in a moving train tosses a coin which falls behind him. Observing this statement what can you say about the motion of the train?
(a) Accelerated
(b) Retarded
(c) Along circular tracks
(d) Uniform

5. Newton's first law of motion says that a moving body should continue to move forever , unless some external forces act on it. But a moving cycle comes to rest after some time if we stop pedaling it. Can you choose the correct reason for the stoppage of cycle?
i. Air resistance
ii. Gravitational pull of the earth

iii. Friction of the road
iv. Heat of the environment
Choose the correct option:
(a) (iii) and (iv)
(b) (i) and (iii)
(c) (i) and (ii)
(d) (ii) and (iii)

6. Newton's third law of motion explains the two forces namely 'action' and 'reaction' coming into action when the two bodies are in contact with each other. These two forces:
(a) Always act on the same body
(b) Always act on the different bodies in opposite directions
(c) Have same magnitude and direction
(d) Acts on either body at normal to each other

7. An object of mass 2 kg is sliding with a constant velocity of 4 m/s on a friction less horizontal table. The force required to keep the object moving with the same velocity is:
(a) 32 N
(b) 0 N
(c) 2 N
(d) 8 N

8 A lift is moving upwards with an acceleration a. A force F exerted by a person of mass m on the floor of the lift is
(a) mg (b) ma (c) m(g + a) (d) m(g - a)

9. Two trains are moving on parallel tracks with the same speed and in the same direction. For a person sitting in one train, the other train appears to be
(a) moving with the same speed (b) at rest
(c) moving with twice the speed of either train (d) none of these

10. A stone dropped from a moving train appears to move along a ....for a person on the platform
(a) straight line (horizontally) (b) parabola (c) vertical straight line
(d) zigzag path

11 The Newton's second law is not applicable if
(a) Force depends on time
(b) Momentum depends on time
(c) Acceleration depends on time
(d) Mass depends on time

12 Which of the following force is not a contact force
(a) Normal force
(b) Frictional force
(c) Gravitational force
(d) Spring force

13 Newton's laws are obeyed in
(a) Accelerated frames of reference
(b) Frame of reference moving with constant velocity
(c) Rotating frames of reference
(d) Frame of reference moving vertically downwards under the action of gravity

14 Which of the following equations represent Galilean transformation equations along the x direction
(a) $x' = x - vt$
(b) $x' = x + vt$
(c) $x' = x - \frac{1}{2}a_i t^2$
(d) $x' = x + \frac{1}{2}a_i t^2$

15 A body of mass 50 kg acquires a speed of 20 ms^{-1} under a force of 100 N in time
(a) 5 s (b) 10 s (3) 15 s (d) 20 s

16 A force of $12N$ gives an object an acceleration of 4 ms^{-2}. The force required to give it an acceleration of 10 ms^{-1} is
(a) 15 N (b). 20 N (c) . 25 N (d). 30 N

Answers : 1 (b) 2 (a) 3 (c) 4 (a) 5(b) 6 (b) 7 (b) 8 (c) 9 (b) 10 (b) 11 (d) 12 (c) 13 (b) 14 (a) 15 (b) 16 (d)

Two mark questions

1. Why is Newton's first called law of inertia? Explain
2. What is the significance of Newton's second law of motion.
3. Action and reaction are equal and opposite, then why they do not cancel each other?
4. What is the significance of Newton's third law of motion.
5. What is frame of reference? Explain.
6. What is inertial frame of reference? Given an example.
7. What is non-inertial frame of reference? Given an example.

7 A body is moving with uniform velocity. Can we say the body is in equilibrium? Explain.
8 Can a body remain at rest even though forces are acting on it? Explain.
9 What are the limitations of Newton's laws of motion?

Six mark questions

1. State and explain Newton' s laws of motion with examples.
2. (a) Distinguish between Inertial and non-inertial frames of reference
   (b) Explain Galilean relativity.
3. Obtain the Galilean transformation equations when two frames are moving with relative uniform velocity.
4 Show that Newton's laws laws of motion are invariant under Galilean transformation.

Friction

Multiple choice questions

1. A block is pulled across a horizontal surface. The mass of the block is 5 kg. The block is travelling at a constant velocity. Calculate the force of friction acting on the block.
(a) 0 N (b) 4 N (c) 15 N (d) 20 N

2. Choose the correct statement
(a) A body can be accelerated by frictional force
(b) There can be zero friction
(c) Kinetic friction is greater than rolling friction
(d) Frictional force and area of contact between the two surfaces are proportional

3 A car moving with speed 72km per hour is to be stopped at shortest distance while moving on a road of coefficient of friction $\mu = 0.5$. The distance is
(a) 30m (b) 40m (c) 72m (d) 20m

4 A block placed on an inclined plane with angle θ, slides down at uniform speed. The coefficient of kinetic friction is
(a) $\sin\theta$ (b) $\cos\theta$ (c)g (d) $\tan\theta$

5 A block of 2kg placed on floor of $\mu = 0.4$ is acted upon by a force of 2.8N parallel to floor. The force of friction between floor and block is
(a) 2.8N (b) 8N (c) 2N (d) zero

6 Use of lubricants cannot reduce
(a) inertia (b) static friction (c) rolling friction (d) limiting friction

7 The relation between the angle of friction and coefficient of friction is
(a) $\mu_s = \tan\theta$ (2) $\mu_s = \sin\theta$ (3) $\mu_s = \cos\theta$ (4) $\mu_s = \cot\theta$

8 An inclined plane makes an angle θ with the horizontal, the force experienced by a body of mass m while sliding down the plane of coefficient of friction μ is
(a) $mg\sin\theta$ (b) $mg(\sin\theta - \mu\cos\theta)$ (c) $mg(\sin\theta + \mu\cos\theta)$ (d) $\mu mg\cos\theta$

9 The acceleration of a body sliding down an inclined plane having coefficient of friction μ_k is

(a) $a = g\,sin\theta\ - \mu_k g\,cos\theta$
(b) $a = g\,sin\theta\ + \mu_k g\,cos\theta$
(c) $a = g\,cos\theta\ - \mu_k g\,sin\theta$
(d) $a = g\,cos\theta\ + \mu_k g\,sin\theta$

10 The static friction is
(a) dependent on the area of contact of the given surfaces when the normal force is constant.
(b) independent of the area of contact of the given surfaces when the normal force is constant.
(c) Less than the kinetic friction
(d) None of the above

11 A force of 10 N acts on a body of mass 2 kg at rest and moves horizontally with acceleration $2ms^{-2}$. The frictional force is
(a) 12 N (b) 8 N (c) 10 N (d) 6 N

12 A body of mass 5 kg slides along the frictionless surface of an inclined plane with the angle of inclination as 30^0 to the horizontal. The acceleration of the body is
(a) 4.9 ms^{-2} (b) 9.8 ms^{-2} (c) 8.477 ms^{-2} (d) 11.2 ms^{-2}

13 The acceleration of two masses 10 kg and 20 kg suspended with the help of a string from a pulley in a Atwood machine is
(a) 4.3 ms^{-2} (b) 5.3 ms^{-2} (c) 3.3 ms^{-2} (d) 2.3 ms^{-2}

14 A mass 3 kg moves on a frictionless table with the help of another mass of 5 kg attached through a string moving over a pully fixed at the edge of the table. The acceleration of the mass is
(a) 4.125 ms^{-2} (b) 5.125 ms^{-2} (c) 3.125 ms^{-2} (d) 6.125 ms^{-2}

15 Which of the following is true ?
(a) $\mu_r < \mu_k < \mu_s$ (b) $\mu_r > \mu_k > \mu_s$ $\mu_r < \mu_k > \mu_s$ $\mu_r > \mu_k < \mu_s$

Answers : 1 (a) 2 (c) 3 (b) 4 (a) 5 (a) 6 (a) 7 (a) 8 (b) 9 (a) 10 (b) 11 (d) 12 (a) 13 (c) 14 (d) 15 (a)

Motion in a resistive medium

Multiple choice questions

1 Drag force depends on
(a) Effective cross-sectional area of the falling object
(b) Density of the surrounding resistive medium
(c) Velocity of the falling body
(d) All the above

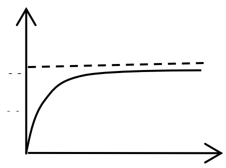
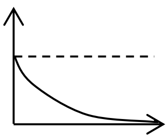
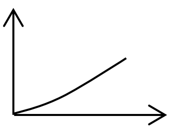
2 Terminal velocity is the constant velocity attained by a body moving under the action of gravity under the condition
(a) Drag force is greater than weight of the body
(b) Drag force is less than weight of the body
(c) Drag force is equal to weight of the body
(d) Drag force and weight equal to each other but acting in the same direction

3 The expression for the terminal velocity when the force is proportional to velocity is
(a) $v_t = \frac{mg}{c}$ (b) $v_t = \sqrt{\frac{mg}{c}}$ (c) $v_t = \frac{c}{mg}$ (d) $v_t = \sqrt{\frac{c}{mg}}$

4 The terminal velocity of a body of mass 2 kg falling through a resistive medium of drag coefficient 0.6 when force is proportional to velocity is $(g = 9.8\, ms^{-2})$
(a) $7.56\, ms^{-1}$ (b) $15.4\, ms^{-1}$ (c) $25.6\, ms^{-1}$ (d) $32.6\, ms^{-1}$

5 The terminal velocity of a body of mass 2 kg falling through a resistive medium of drag coefficient 0.6 when force is proportional to square of velocity is $(g = 9.8\, ms^{-2})$
(a) $12.52\, ms^{-1}$ (b) $15.4\, ms^{-1}$ (c) $5.72\, ms^{-1}$ (d) $32.6\, ms^{-1}$

6 The terminal velocity of a parachute of mass 90 kg with cross sectional area 30 m^2, moving a resistive medium of density $1.25\, kgm^{-3}$ and drag coefficient 1.3 is
(a) $22\, ms^{-1}$ (b) $6.1\, ms^{-1}$ (c) $9.4\, ms^{-1}$ (d) $12.5\, ms^{-1}$

7 Which of the following graph represent the variation of instantaneous velocity with time of a body moving in a resistive medium under the action of gravity when the force is proportional to the velocity is
(a)

(b)

(c)

(d) None of these

8 A 150 g pebble is falling with a constant terminal velocity of 30 ms^{-1}. The drag coefficient is
(a) 0.049 (b) 0.059 (c) 0.069 (d) 0.079

9 The drag coefficient of a 75 kg skydiver with a terminal velocity of 60 ms^{-1} is
(a) 0.204 (b) 0.045 (c) 0.45 (d) 0.65

10 By how much would the skydiver need to reduce his projected area so as to double his terminal velocity
(a) By half (b) by one fourth (c) by one sixth (c) by one eighth

Answers : 1 (d) 2 (c) 3 (a) 4 (d) 5 (c) 6 (b) 7 (a) 8 (a) 9 (a) 10 (b)

Two mark questions

1 What is Drag force? Mention a factor on which it depends.
2. Explain terminal velocity

3. What are the different forces acting on a body moving vertically through a resistive medium under the action of gravity?
4 What is drag coefficient? Mention the factors on which it depends.
5 Why does a body move with non-uniform acceleration when dropped vertically in a resistive medium?
6 The terminal velocity of a body moving vertically downwards in water is less than that when moving in air. Why?

Six mark questions

1 Arrive at the expressions for terminal velocity when (i) force is proportional to velocity and (ii) force proportional to square of velocity.
2 Arrive at the expression for velocity of a body falling freely under gravity where the resistance varies directly as velocity of the body. Represent the variation graphically
3 Given the velocity of a body moving in a resistive medium under the action of gravity as $\boldsymbol{v} = \boldsymbol{v_t}\left(\mathbf{1} - \boldsymbol{e}^{-\frac{c}{m}t}\right)$, arrive at the expressions for the acceleration and displacement of the body. Represent the variations graphically.
4 Arrive at the expression for terminal velocity in terms of density and area of cross section of a body moving under action of gravity in case of force proportional to square of velocity.

Rotational frames of reference

Multiple choice questions

1. The expression for Coriolis force is
(a) $-2m(\overrightarrow{\omega} \times \vec{v}')$ (b) $-m\omega \times (\omega \times \vec{r})$ (c) $-(\omega \times \vec{r})$ (d) $-(\omega \times \vec{v}')$

2. The horizontal deflection effect caused by the Coriolis Force is
(a) Greater near the pole
(b) Greater near the equator
(c) Equally Distributed all over the earth's surface
(d) More near the surface of the earth and lesser above

3 Which of the following is correct about Coriolis Force?
(a) Coriolis Force affects only the large-scale movement of air in the atmosphere
(b) Coriolis Effect is caused due to rotation of Earth and pressure difference between two points
(c) Winds are deflected to the right in the Southern hemisphere
(d) If it was not for the Earth's rotation, global winds would blow in straight north-south lines

4 Which of these is not a major cause of global wind circulation?
(a) Inequalities in the distribution of solar radiation
(b) The Coriolis effect
(c) Position of the major earth mountain ranges
(d) Carbon dioxide concentration

5 Which of the following is not correct for a non inertial frame of reference
(a) velocity does not remain constant.
(b) Newtons laws are obeyed
(c) Fictitious forces arise
(d) Zero force does not correspond to zero acceleration

6 The effect of centrifugal force due to rotation of earth
(a) Maximum at the poles
(b) Maximum at the equator
(c) Effect is same at the equator and the poles
(d) No changes in centrifugal force

7 Which of the following is not correct
(a) Centrifugal force exists only when viewed from the rotating frame.
(b) Centrifugal force has no existence in an inertial frame.
(c) Coriolis force always acts at right angles to the direction of motion of the particle, and does work.
(d) It only changes the direction of motion of the particle without changing the speed

8 Which of the following is not correct for Coriolis force?
(a) Coriolis force is zero. when the angular velocity parallel to the particle velocity
(b) Coriolis force is maximum when the angular velocity is at right angles to particle velocity
(c) Coriolis force is zero if the particle is at rest in a rotating frame
(d) Coriolis force is independent of rotation of earth

9 Due to the rotation of the earth, the value of g at latitude λ decreases by a
(a) $\omega^2 R cos^2\lambda$ (b) $\omega^2 R sin^2\lambda$ (c) $\omega^2 cos^2\lambda$ (d) $\omega^2 sin\lambda$

10 The plane of oscillation of a Foucault pendulum at the poles makes a complete revolution in
(a) 24 hours (b) 12 hours (c) 18 hours (d) 6 hours

Answers : 1 (a) 2 (a) 3 (d) 4 (d) 5 (b) 6 (b) 7 (c) 8 (d) 9 (a) 10 (a)

Two mark questions

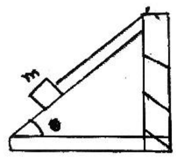
1 What is non inertial frame of reference. Give an example.
2 Is Newton's second law of motion valid in a rotating frame? Explain
3 What is meant by frame dependent force? Explain with an example.
4 What is Coriolis force? Explain.
5 What is Centrifugal force? Explain
6 Why the outer edge of a railway track or outer edge of a road slightly elevated than the inner one?
7 Ia earth an inertial frame? Explain.

8 What are the forces that come into play in a rotating frame?
9 Does the Coriolis force do work? Explain.
10 What will be the direction of Coriolis force in the northern and southern hemisphere?
11 How do you account for the greater erosion to one bank of a river than the other.
12 Explain the formation of trade winds
13 Explain the formation of cyclones.

Six mark questions

1. Prove that a frame of reference moving with uniform acceleration in translatory motion with respect to an inertial frame is a non inertial frame.
2 Arrive at the expressions for Coriolis force and centrifugal force acting on a particle in a rotational frame of reference.
3 Explain the two fictitious force acting in a rotational frame namely Coriolis force and centrifugal force.
4 Explain the physical significance of the terms in the equation $\vec{F'} = \vec{F} - 2m(\vec{\omega} \times \vec{v}') - m\omega \times (\omega \times \vec{r})$
5. Assuming the earth to be a rotating frame of reference, explain the effect of centrifugal force on the value of g. Also show that the effect is maximum at the equator and minimu at the poles.
6 Explain the effect of Coriolis force on a moving particle on the surface of earth. Calculate the horizontal and vertical components of Coriolis force.

Numerical problems



1 The diagram shows a block of mass m = 15 kg held by a chord on a frictionless inclined plane. What is the tension in the chord if $\phi = 30^0$. What force does the plane exert on the block?

$mg\ sin\theta - T = F = 0 \quad or \quad T = mg\ sin\theta = 73.5\ N \quad and \quad N = mg\ cos\theta = 127.15\ N$

2 A block is pulled at a constant velocity on a horizontal surface by a force of 10N applied at an angle of 25^0.to the horizontal. If the coefficient of friction is 0.5, find the mass of the block.

As the velocity is constant, the applied force is equal to kinetic frictional force.

$F = f_k \quad As \quad f_k = \mu_k N = \mu_k mg \quad$ Thus $\quad m = \frac{f_k}{\mu_k g} = 2.041\ kg$

3 A block of mass 1 kg is placed on a horizontal surface. The coefficient friction between the block and the surface is 0.2. If an external force of 3 N is applied on the block parallel to the surface, find the acceleration of the block.

$F_{net} = F - f = ma \quad As\ f = \mu_s N = \mu_s mg$ Thus $a = \frac{F-f}{m} = \frac{F - \mu_s mg}{m} = 1.04\ ms^{-2}$

4 A 10 kg block is placed on a horizontal table and is attached to a 5 kg block with the help of a string passing over a frictionless massless pully. Calculate the acceleration produced. Given the coefficient of static friction is 0.2 between block and the table.

$a = \frac{m_2 g - \mu_s m_1 g}{m_1 + m_2} = 1.96\, ms^{-2}$

5 Two masses 6 kg and 12 kg are connected at the two ends of a light inextensible string that goes over a frictionless pulley. Find the acceleration of the masses when masses are released.

$a = \frac{(m_2 - m_1)g}{m_1 + m_2} = 3.26\, ms^{-2}$

6. Calculate the fictitious and total force acting on a freely falling body of mass 18 kg with respect to frame moving with a downward acceleration of 6 ms^{-2}.

Fictitious force $F_0 = ma_i = 18 \times 6 = 108\, N$

Total force $F' = F - ma_i = mg - ma_i = 68.4\, N$

7. Three blocks of the same mass $m = 1kg$ each, connected by cords are pulled by a force F on a smooth horizontal surface as shown. Find the tensions T_1, T_2 and the force F if the acceleration is 10 ms^{-2}

$T_1 = ma, \quad T_2 - T_1 = ma, \quad F - T_2 = ma \qquad T_1 = 10\, N, T_2 = 20\, N \;\; T_3 = 30\, N$

8. Find the acceleration of a block 1 and tension in the string shown in the diagram if the coefficient of kinetic friction is 0.2. Given $m_1 = 5\, kg$, $m_2 = 10\, kg$, $g = 10\, ms^{-2}$, $\theta = 30^0$.

$a = \frac{m_2 g - \mu_k m_1 g cos\theta - m_1 g sin\theta}{m_1 + m_2} = 4.84 ms^{-2} \qquad T = m_2 g - m_2 a = 49.6\, N$

9. A raindrop with radius 1.2mm falls from a cloud that is at a height of 1.2 km above ground. The drag coefficient is 0.6. Calculate the terminal speed of the drop if density of water is 1000 kgm^{-3} and density of air is 1.23 kg m^{-3}.

$v_t = \sqrt{\frac{2mg}{C\rho A}} = 3.18\, ms^{-1}$

10. An object is dropped in a viscous medium in which resistance varies directly as velocity of the object. How long does it take for the object to reach one – fourth of its terminal velocity? Given : Mass of the object = 100 g and drag coefficient = 0.2 Nsm^{-1}. Also, calculate the terminal velocity.

$v = v_t \left(1 - e^{-\frac{c}{m}t}\right) \qquad v_t = 4.9\, ms^{-1}$

11. Calculate the magnitude and direction of Coriolis acceleration of a rocket moving vertically upward with a velocity of 2 kms^{-1} at $60^0 S$ latitude. ($\omega = 7.3 \times 10^{-5} rads^{-1}$)

$a = 2\omega\, v_x cos\lambda = 2 \times 7.3 \times 10^{-5} \times 2 \times 10^3 \times \cos 60 = 14.6\ \times 10^{-2} ms^{-2}$ towards east

12. Calculate the Coriolis force acting on a body of mass 5 kg moving with velocity 2 kms^{-1} on the earth's surface with the direction of angular velocity of earth and direction of the body are at right angles to each other. ($\omega = 7.3 \times 10^{-5} rads^{-1}$)
$F = 2m(\vec{\omega} \times \vec{v}') = 2m\omega v\ sin\theta = 2 \times 5 \times 7.3 \times 10^{-5} \times 2 \times 10^3 \times sin 90 = 146 \times 10^{-2} N$

13. Calculate the fictitious force and the total force acting on a mass of 3 kg in a frame of reference moving (i) vertically downwards and (ii) vertically upwards with an acceleration of $4ms^{-2}$. The acceleration due to gravity is $9.8ms^{-2}$.
(i) For downward motion, the fictitious force $F_0 = -ma_0 = -3 \times (-4) = 12\ N\ (upwards)$
Thus, total force $F' = F + F_0 = -29.4 + 12 = -17.4\ N\ (downwards)$
(ii) For upward motion, the fictitious force $F_0 = -ma_0 = -3 \times (+4) = -12\ N\ (upwards)$
Thus, total force $F' = F + F_0 = -29.4 - 12 = -41.4\ N\ (downwards)$

14 A mass of 1 kg is hurled horizontally due north with a velocity of 0.5 kms^{-1} at a latitude of $30^0 N$. Find the magnitude and direction of the Coriolis force acting on the mass.
$F_x = 2m\omega v sin\lambda = 2 \times 1 \times 7.3 \times 10^{-5} \times 0.5 \times 10^3 \times \sin 30 = 3.65 \times 10^{-2}\ N, direction\ 60^0\ with\ east$

15 A 200 kg projectile is fired due east with initial speed of $500ms^{-1}$. If the latitude of the place is $50^0 N$, find the magnitude of Coriolis force.
$F_x = 2m\omega v sin\lambda = 2 \times 200 \times 7.3 \times 10^{-5} \times 500 \times \sin 50 = 11.18\ N$

16 Find the latitude of the place where the plane of oscillation of Foucault's pendulum rotates once in a day
$T = \frac{2\pi}{\omega\ sin\lambda}$ Here T = 24 hours and $\frac{2\pi}{\omega} = 24\ hours.\ Thus\ 24 = 24 sin\lambda\ \ or\ \ sin\lambda = 1\ Thus\ \lambda = 90^0$ at the poles.