Model Paper - I

Mathematics VII

I Answer any 15 questions :- ($15 \times 2 = 30$)

1. Let V (F) be a vector space over a field F then prove that c. 0 = 0 where $c \in F$ and $0 \in V$.
2. Define a subspace of a vector space.
3. Prove that the set $s = \{(1,2,1),(2,1,1),(1,-1,2)\}$ is linearly independent.
4. Show that $3x^2 + x + 5$ polynomial is the linear span of the set $s = \{x^3, x^2 + 2x, \quad x^2 + 2, \quad 1 - x\}$
5. Define $T : V_2(R) \to V_2(R)$ by $T(x, y) = (x\cos\theta - y\sin\theta,$ $x\sin\theta + y\cos\theta)$. Verify whether T is a linear transformation.
6. Find the matrix of the linear transformation $T : R^3 \to R^2$ defined by $T(x, y, z) = (2x + 3y, y + 2z)$ w.r.t standard bases.
7. Let $T : V_2(R) \to V_2(R)$ be defined by $T(1,0) = (1,2)$ $T(0,1) = (4,3)$. Find all eigen values of T.
8. Evaluate $\int (3x - 2y)dx + (y + 2z)dy - x^2 dz$ where x = t, $y = 2t^2$, $z = 3t^3$ varying from 0 to 1.
9. Prove that $\int_0^{\pi/2} \int_0^{a\cos\theta} r^2 d\theta\, dr = \frac{2a^3}{9}$
10. Evaluate $\iint_D xy\, dx\, dy$ over the rectangle bounded by $x = 2$, $x = 5, \quad y = 1 \quad and \quad y = 2$.
11. Prove that $\int_0^1 \int_0^2 \int_1^2 x^2 y\, z\, dz\, dy\, dx = 1$.

12. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration.
13. State Stokes theorem in a plane.
14. Using Green's theorem evaluate $\int (x^2 y\cos x + 2xy\sin x - y^2 e^x)dx + (x^2 \sin x - 2ye^x)dy$ around the hypocycloid $x^{2/3} + y^{2/3} = a^{2/3}$.
15. Using Divergence theorem, show that $(axi + byj + (czk).\hat{n}\, ds = \frac{4}{3}\pi(a + b + c)$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$.
16. If $\overline{f}(t) = (3t^2 - t)i + (2 - 6t)\hat{j} - 4tk$, find $\int_2^4 \overline{f}(t)\,dt$
17. The acceleration of a particle at t is given by $4\cos 2t\hat{i} - 8\sin 2t\hat{j} + 16t\hat{k}$, find its displacement at any time t.
18. Show that the Euler's equation for the extremum of $\int_{x_1}^{x_2} (y^2 + y'^2 + 2ye^x)dx$ reduce to $y'' - y = e^x$.
19. State the necessary condition for $I = \int_{x_1}^{x_2} f(x, y, y')dx$ to be an extremum.
20. Obtain the general solution of the extremal problem $\int_a^b \frac{(y')^2}{x^3}dx$ when $b > a > 0$.

II Answer any four of the following :- ($4 \times 5 = 20$)

1. Show that the set $V = \left\{\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} \middle| x, y \in R\right\}$ is a vector space over the field of reals R under + and $\times$ of matrices .

2. Prove that a non – empty subset w of a vector space V (F) is a subspace of V (F) iff $\forall \alpha, \beta \in w$ and for any $a, b \in F$, $a\alpha + b\beta \in w$
3. Find the dimension and basis of the subspace spanned by $(2,4,2)\,(1,-1,0)\,(1,2,1)$ *and* $(0,3,1)$ *in* $V_3(R)$.
4. State and prove Rank – Nullity theorem of a linear transformation
5. Find a linear transformation $T: V_2(R) \to V_2(R)$ such that $T(1,2) = (3,0)$ *and* $T(2,1) = (1,2)$
6. Find the Eigen space of the linear transformation $T: R^3 \to R^3$ defined by $T(x,y,z) = (2x+y,\, y-z,\, 2y+4z)$.

III. Answer any three of the following : (3 × 5 = 15)

1. Evaluate $\int (x+2y)\,dx + (4-2x)\,dy$ around the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ in the anti clockwise.
2. If C is a curve leading from $(0,0,1)$ *to* $\left(1, \frac{\pi}{4}, 2\right)$ show that
$$\int 2xyx^2 dx + \left(x^2z^2 + Z\cos yz\right)dy + \left(2x^2yz + y\cos yz\right)dz = 1 + \pi$$
3. Evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$ by changing the order of integration.

OR

Evaluate $\iiint dxdydz$ taken through the region defined by x, y, z ≥ 0, $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} \le 1$

4. Evaluate $\int_0^a \int_0^{\sqrt{a^2-z^2}} \int_0^{\sqrt{a^2-y^2-z^2}} x\,dx\,dy\,dz$

OR

Find the volume of the tetrahedran bounded by the co-ordinates planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

IV Answer any three of the following : - (3 × 5 = 15)

1. If $\int_c F.dr$ from $(0,0)$ *to* $(1,1)$ along the paths C a) The parabola y = x^2 b) the line from (0, 0) to (1, 0) and then to (1, 1,)
2. Evaluate $\int\int_C\int \phi\, dv$ where $\phi = 45x^2y$ and C is the close region bounded by the planes $4x + 2y + z = 8$ $x = 0,\, y = 0,\, z = 0$.
3. State and prove Green's theorem in a plane.
4. Evaluate $\int_S\int \left[(x+z)\hat{i} + (y+z)\hat{j} + (x+y)\hat{k}\right].\hat{n}\, ds$ where S is the surface pf the sphere $x^2 + y^2 + z^2 = 4$ by using Gauss – divergence theorem.

5. Evaluate $\int_c \left(xy\,dx + xy^2\,dy\right)$ by Stokes theorem where C is the square in xy – plane with vertice $(1,0),(-1,0),(0,1)\ and\ (0,-1)$.

V **Answer any two of the following :-** **(2 × 5 = 10)**

1. Show that the curve passing through $(1,0)\, and\, (2,1)$ with $\int_1^2 \sqrt{\frac{1+(y')^2}{x^2}}\,dx$ is a circle.
2. Find the extremum of the functional $\int_{x_1}^{x_2} \left[y^2 + (y')^2 + 2y\sin 2x\right]dx$
3. Find the geodesics on a right circular cone.
4. Find the external of the functional $\int_0^1 \left[x^2 + \lambda y^2 - (y')^2\right]dx$ under the conditions $(0,0)(1,0)$ and subject to the constraint $\int_0^1 y^2 dx = 2$.

Model Paper II
Mathematics VII

I. **Answer any 15 questions :-** **(15 × 2 = 30)**

1. Define a vector space over the field $\left(F + \square\right)$

2. Prove that the subset $w = \{(x,y,z) \mid x - 3y + 4z = 0\}$ of the vector space R^3 os a subspace of R^3.
3. Show that $(2,-1,-8)$ vector can be expressed as a linear combination of the vectors $(1,2,1),(1,1,-1),(4,5,-2)$.
4. Show that the vectors $(1,0,-1)(1,2,1)(0,-3,2)$ form a basis of $V_3(R)$.
5. Verify whether $T: R^3 \to R^2$ defined by $T(x,y,z) = (2x+3y,\quad y+z+1)$ is a linear transformation.
6. Find the matrix of the linear transformation $T: R^2 \to R^3$ defined by $T(x,y) = (x+y,\quad y+z,\quad z+x)$.
7. Find the eigen values of the linear transformation $T: R^3 \to R^3$ defined by
$T(e_1) = (4,0,1)\, T(e_2) = (-2,1,0)\, T(e_3) = (-2,0,1)$
8. Evaluate $\int (x+y)\,dx + x^2 y dy$ along the straight line $y = 3x$ from $(0,0)\ to\ (3,9)$.
9. Prove that $\int_0^1\int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}} = \frac{\pi^2}{4}$.
10. Evaluate $\iint dx\,dy$ over the area bounded by $x = 0,\ y = 0,$ $x^2 + y^2 = 1\quad and\quad y = \frac{1}{2}$.
11. Evaluate $\int_0^1\int_0^1\int_0^1 (x+y-z)\,dydx$ dz.
12. Evaluate $\int_0^{2a}\int_0^{b} (x^2+y^2)\,dydx$ by changing into polar coordinates.
13. State Divergence theorem in a plane.

14. Using Green's theorem find the area of the circle $x^2 + y^2 = a^2$
15. Using stokes theorem prove that $\oint_c \bar{r} \circ d\bar{r} = 0$
16. If $\bar{r} = 5t^2\hat{i} + t\hat{j} - t^3 k$ then prove that $\int_1^2 \bar{r} \times \frac{d^2\bar{r}}{dt^2} dt = -14\hat{i} + 75\hat{j} - 15k$.
17. If $\vec{F} = y\hat{i} - x\hat{j}$, evaluate $\int_c \vec{F}.d\vec{r}$ along the parabola y = x^2 from (0,0) to (1,1)
18. Prove that the Euler's equation for the extremum of $\int_1^2 \left[x^2 (y')^2 + 2y(x+y) \right] dx = 0$ is $x^2 y'' + 2xy' - 2y = x$
19. Prove that the extremal of $\int_0^1 \frac{(y')^2}{x} dx$ with $y(0) = 0$, $y(2) = 1$ is a parabola.
20. Briefly explain Isoperimetric problems .

II Answer any four of the following :- (4 × 5 = 20)

1. Show that $V = \left\{ a + b\sqrt{2} \middle| a, b \in Q \right\}$ where Q is the field of all rationals forms a vector space under usual + and ×.
2. Prove that A non – empty subset w of a vector space V is a subspace of V iff (i) $\forall \alpha, \beta \in w \Rightarrow \alpha + \beta \in w$ (ii) $c \in F, \alpha \in w \Rightarrow c\alpha \in w$.
3. Define basis and dimension of a vector space. Find the basis and dimension of subspace of $V_3(R)$ spanned by $\{(1,-2,3), \ (1,-34) \ (-1,1,-2)\}$.

4. Find the matrix of the linear transformation $T : R^2 \to R^3$ defined by $T(x, y) = (2y - x, y, 3y - 3x)$ relative to bases $B_1 = \{(1,1), (-1,1)\}$ and $B_2 = \{(1,1,1), (1,-1,1)(0,0,1)\}$
5. If $T : V_3(R) \to V_4(R)$ defined by $T(e_1) = (0,1,0,2)$ $T(e_2) = (0,1,1,0), T(e_3) = (0,1,-1,4)$. Find the range space null space, rank and nullity of T and verify the rank and nullity theorem.
6. Find the eigen values and eigen vectors of the linear transformation $T : R^3 \to R^3$ given by $T(x, y, z) = (x + 3z, 2x + y - z, x - y + z)$

III Answer any three of the following :- (3 × 5 = 15)

1. Prove that the value of the integral $\int_c (x^2 - y^2)dx + x^3 y dy = 56\pi$ where c is the semi circle with centre (0, 4) and radius 2 units.
2. Evaluate $\int_c F.dr$ where $\bar{F} = e^x \sin y\hat{i} + e^x \cos y\hat{j}$ and c is the rectangle with vertices $(0,0)(1,0)\left(1, \frac{\pi}{2}\right)\left(0, \frac{\pi}{2}\right)$.
3. Prove that $\iint_R xy(x+y)\,dx\,dy = \frac{3}{56}$ over the region R between y = x^2 and y = x .
4. Evaluate $\iiint dxdydz$ where v is the volume of the tetrahedron whose vertices are (0, 0, 0), (0, 1, 0) 91, 0, 0

   OR

   Evaluate $\int_0^{\pi/2} \int_0^{a\sin\theta} \int_0^{(a^2-r^2)/a} rdz.dr.d\theta$
5. Using triple integral, find the volume of the sphere.

IV Answer any three of the following :- **($3 \times 5 = 15$)**

1. Find the work done in moving a particle once round the circle $x^2 + y^2 = 4$ in a force field $F = (2x - y + 2z)\hat{i} + (x + y - z)\hat{j} + (3x - 2y - 5z)k$.
2. Evaluate $\int (x^2 - 2xy)dx + (x^2 y + 3)\, dy$ using Green's theorem around the curve $y^2 = 8x$ and $x = 2$.
3. Verify Gauss divergence theorem for $F = y\hat{i} + x\hat{j} + z^2 k$ over the cylindrical region S bounded by $x^2 + y^2 = a^2$, $z = 0$ *and* $z = h$.
4. Verify Stokes theorem for $\overline{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken round the rectangle bounded by $x = \pm a, \quad y = 0, \quad y = b$
5. Show that $\int_{(0,1)}^{(1,2)} (x^2 + y^2)dx + 2xydy$ is independent of the path and find its value.

V Answer any two of the following:- **(2 x 5 = 10)**

1. Show that the external value of $\int_0^1 \left[x + y + \left(y^1\right)^2\right] dx$ with y(0) = 1**,** y(1) = 2 is $4y = x^2 + 3x + 4$**.**
2. If a cable hangs freely under gravity from two fixed points then show that the shape of the curve is a catenary.
3. Show that the geodesic of a sphere of radius r and its great circles.

4. Find the extremal of the functional $\int_0^{\pi} \left[(y^1)^2 - y^2\right] dx$ under the condition (0, 0) and (π, 1) and subjected to the constraint $\int_0^{\pi} ydx = 1$.

MODEL PAPER III
MATHEMTICS VII

I Answer any 15 questions:- **(15 x 2 = 30)**

1. Let V(F) be a vector space over a field F. Prove that c(α - β) = cα - cβ $\forall c \in F$ and $\alpha, \beta \in V$.
2. What are the criterion for a subset to be a subspace.
3. Define Linear combination on a vector space V(F)
4. Determine whether the set S = {(2, 1) (1, -2) (1, 0)} is a basis of R^2.
5. Prove that if $T : v_3(R) \to v_2(R)$ is defined by T(x, y, z) = (x + z, y – x + z) is a linear transformation .
6. If T: $v_3(R) \to v_2(R)$ defined by $T(x, y, z) = (y - x, y - z)$. Find matrix of T.
7. Find the eigen values of Linear transformation $T(e_1) = (1, 4)$, $T(e_2) = (2, 3)$ where T : v_2 (R) → v_2(R).
8. Evaluate $\int_c (3x^2 + 6y)dx - 14yzdz + 20x^2 z\, dz$ where c is the curve x= t , y = t^2, z = t^3 from the origin to (1, 1, 1)
9. Prove that $\int_0^1 \int_0^{1-x} (x^2 + y^2)dxdy = \frac{1}{6}$.
10. Evaluate $\iint_D xy\, dx\, dy$ where D is the region bounded by x = 0, y = 2 and x^2 = 2y.

11. Evaluate $\int_0^3\int_0^2\int_0^1 (x+y+z)\,dz\,dy\,dx.$
12. By changing into polar coordinates evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$
13. State Greens theorem in a plane.
14. Using Divergence theorem, prove that $\iint_s \bar{r}.\hat{n}ds = 3v$ where s is the closed surface.
15. Using stokes theorem, Show that $\oint_c (yzdx + xzdy + xydz) = 0.$
16. The acceleration of a particle at any time t is given by $\bar{a} = \bar{e}^{t}\,\hat{i} - 6(t+1)\hat{j} + 3\sin t\,\hat{k}$. Find its velocity when t = 0 $\bar{v} = 0$.
17. If $\vec{f} = \sin u\hat{i} + \cos u\hat{j} + u^2 k$ find $\int \vec{f} \circ \frac{d\vec{f}}{du}du$
18. Obtain the Euler's equation to the external of $\int_{x_1}^{x_2}\left[y^2 + (xy')^2 + ye^x\right]dx$
19. Show that the extremal of $\int_{x_1}^{x_2}\sqrt{1+(y')^2}\,dx$
20. Show that the geodesic on a plane is a straight line.

II Answer any four of the following :- (4 × 5 = 20)

1. Show that the set of all matrices of the order n × n with their elements as real numbers is a vector space over the field $(R+,\cdot)$ with the usual operations of matrices.
2. Show that the intersection of any two subspaces of a vector space V over a field F is a subspace of V. Also show that union of two subspaces need not be a subspace.

3. Find the dimension and basis of the subspace spanned by $(1,3,2,4)$, $(1,5,-2,4)(1,2,3,4)(1,6,-3,4)$ $in\ V_4(R)$
4. Find the linear transformation for the matrix $A = \begin{bmatrix} -1 & 0 \\ 2 & 0 \\ 1 & 3 \end{bmatrix}$ w.r.t $B_1 = \{(1,2,0),(0,-1,0)(1,-1,1)\}$ and $B_2\{(1,0),(2,-1)\}$
5. Let $T : R^3 \to R^3$ definedby $T(x,y,z) = (x+y, x-y, 2x+z)$. Verify the rank and nullity theorem.
6. Find the Eigen space of the L.T $T : R^3 \to R^3$ defined by $T(x,y,z) = (3x, 2y+z, -5y-2z)$

III Answer any threee of the following :- (3 × 5 = 15)

1. Evaluate $\int (x+y)dx + (y-x)dy$
   (i) along the parabola $y^2 = x$ from (1,1) to (4,2)
   (ii) along $x = 2t^2 + t + 1, \quad y = t^2 + 1 \quad at \quad t = 1$
2. Evaluate $\int_0^3\int_1^{\sqrt{4-y}} (x+y)\,dxdy$ by changing the order of integration.
3. Evaluate $\iiint xyz \quad dx\,dy\,dz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$

OR

Evaluate $\int_0^1\int_0^{1-x}\int_0^{1-x-y}\frac{dzdydx}{(1+x+y+z)^3}$

4. By changing into polar coordinates evaluate $\iint_R\sqrt{x^2+y^2}\,dxdy$ where R is the circle $x^2+y^2=a^2$

5. Find the surface area of the sphere $x^2+y^2+z^2=a^2$

IV Answer any three of the following :- ($3\times5=15$)

1. If $\overline{F}=\left(2xy^3-y^2\cos x\right)\hat{i}+\left(1-2xy\sin x+3x^2y^2\right)\hat{j}$ find the value of $\int_c F.dr$ along the curve C given by $2x=\pi y^2$ from $(0,0)\ to\left(\pi/2,1\right)$.

2. Evaluate $\iint F.\hat{n}ds$ where $F=y\hat{i}+2x\hat{j}-z\hat{k}$ in the surface of the plane 2x + y = 6 in the first octant cut off by plane z = 4.

3. Verify Green's theorem for $\oint_c\left(3x^2-8y^2\right)dx+\left(4y-6xy\right)dy$ Where C is the boundary of the region $y=\sqrt{x},\quad y=x^2$.

4. Evaluate $\iint_s F.\hat{n}\,ds$ where $F=4xy\hat{i}+yz\hat{j}-xzk$ and S is the Surface of the cube bounded by the planes $0\le x\le 2$, $0\le y\le 2,\ 0\le z\le 2$.

5. Verify stokes theorem for the function $F=y^2\hat{i}+xy\,\hat{j}-xz\hat{k}$ Where S is the hemisphere $x^2+y^2+z^2=a^2,\quad x\ge 0$.

V. Answer any two of the following :- ($2\times5=10$)

1. Show that the extremal value of $\int_1^2 x^2\left(y'\right)^2dx$ with $y(0)=1$, $y(2)=1\ is\ y=\frac{4}{3}\left(1-\frac{1}{x^2}\right)$.

2. Prove that the extremal of $\int_{x_1}^{x_2} y\sqrt{1+\left(y'\right)^2}\,dx$ is the catenary $y=a\cosh\left(ax+b\right)$.

3. Show that the shortest distance between two points in a plane is along the straight line joining them

4. Find the extremal of the function $\int_0^1\left[\left(y'\right)^2+x^2+\lambda y\right]dx$ under the condition $y(0)=0,\quad y(1)=0$ and subjected to the constraint $\int_0^1 ydx=\frac{1}{6}$.