Syllabus : Paper 401 – Unit III : Lasers - Introduction; Spontaneous and stimulated emission; Einstein's coefficients and optical amplification; Population inversion; Main components of a laser; Lasing action; Ruby Laser - construction and working - energy level diagram; He-Ne Laser - construction and working - energy level diagram
Spatial Coherence and directionality, estimates of beam intensity, temporal coherence and spectral energy density

Introduction : The word **LASER** was coined as an acronym for **Light Amplification by Stimulated Emission of Radiation**.

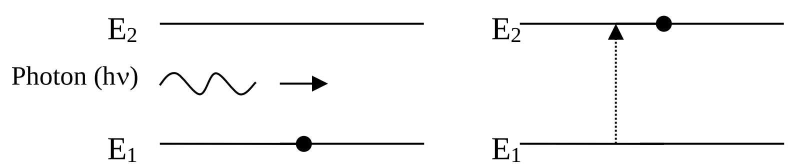
The special nature of laser light has made laser technology a vital tool in nearly every aspect of everyday life including communications, entertainment, manufacturing, and medicine. A **laser device** is a source of highly intense, monochromatic, coherent and highly directed beam of light produced by stimulated emission of radiation. **T. Maiman** produced first laser device using **ruby crystal** in 1960. The idea of stimulated emission of radiation was given by Albert Einstein in 1917.

Interaction between matter and electromagnetic radiation

An isolated atom can exist in its ground state of energy E_1 or in an excited state of higher energy E_2. The atom can change from one of these states to the other through following three processes. They are

1. Stimulated absorption, 2. Spontaneous emission and 3. Stimulated emission.

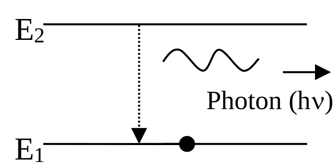
Stimulated absorption



Consider an atom initially in the ground state of energy E_1. If an electromagnetic radiation of frequency ν is incident on the atom, the atom absorbs energy $h\nu$ from the radiation and move to the higher energy state E_2 if $h\nu = E_2 - E_1$.

The process by which an atom in a lower energy state can be raised to a higher energy state by absorbing a photon of energy hν is called **induced or stimulated absorption**..

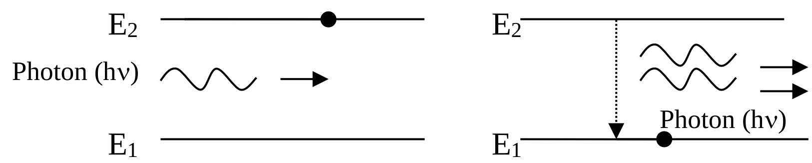
ATOM + PHOTON ⟶ ATOM*



Spontaneous emission : Consider the atom in the excited state of energy E_2. The atom will remain in this state for a time of about $10^{-8} s$. The atom will fall on its own to the ground state emitting radiation of energy hν. The process by which an excited atom jumps from a higher energy state to a lower energy state with emission of a photon is called **spontaneous emission**.

ATOM* ⟶ ATOM + PHOTON

Stimulated emission : Consider an atom in the excited state of energy E_2.
If an electromagnetic radiation of frequency ν is incident on the atom, the atom moves to the ground state of energy E_1 by emitting a photon of energy hν along with the incident photon of energy hν. This stimulated emission is possible only if the frequency ν of the radiation satisfies the relation $h\nu = E_2 - E_1$.



The process in which an atom in a higher energy state jumps to lower energy state with the emission of a photon identical to the incident photon is called as **stimulated emission**.

ATOM* + PHOTON ⟶ ATOM + 2 PHOTONS

Differences between Spontaneous and Stimulated emission

	Spontaneous emission	Stimulated emission
1	Instantaneous emission of light photon due to transition of an atom from higher to lower energy level	Emission of light photon with the help of another photon having energy same as the emitting photon due to transition of atom from higher to lower energy level
2	The emission has broad spectrum of radiation (**many wavelengths**)	The emission has monochromatic radiation (**single wavelength**)
3	Incoherent radiation (light photons of **different phases and different frequencies**)	Coherent radiation (light photons of **same phase and same frequency**)
4	The light beam has **less intensity**	The light beam has **high intensity**
5	**Less directionality and more angular spread.** e.g. Light from sodium or mercury lamp	**High directionality and less angular spread** e.g. Light from a laser source

Relation between Einstein coefficients

Consider an atomic system present in a radiation field of energy density $\rho(\nu)$ under equilibrium conditions. Let E_1 and E_2 be the energy levels of the atom with $E_2 > E_1$. The number of atoms per unit volume in the two levels respectively are N_1 and N_2. The photon emitted due to transition between the levels will have energy $h\nu = E_2 - E_1$.
Postulations made by Einstein are

1 The rate of induced/stimulated absorption of radiation by atoms in the ground state is proportional to (a) the number of atoms per unit volume in the lower level $N_1$and (b) density $\rho(\nu)$of radiation energy incident on these atoms. Mathematically $\frac{dN_1}{dt} \propto N_1\rho(\nu)$

or $\frac{dN_1}{dt} = B_{12}N_1\rho(\nu)$.....(1) where B_{12} is the proportionality constant called Einstein coefficient for induced absorption.

2 The rate of spontaneous emission of radiation by atoms due to transition from higher to lower level is proportional to the number of atoms per unit volume in the higher level N_2 only. i.e. $\frac{dN_2}{dt} \propto N_2$ or $\frac{dN_2}{dt} = A_{21}N_2$.....(2) where A_{21} is the proportionality constant called Einstein coefficient for spontaneous emission.

3 The rate of stimulated emission of radiation by atoms due to transition from higher to lower level is proportional to (1) the number of atoms per unit volume in the higher level N_2 and (2) density of radiation energy incident on these atoms. i.e. $\frac{dN_2}{dt} \propto N_2\rho(\nu)$

or $\frac{dN_2}{dt} = B_{21}N_2\rho(\nu)$.....(3) where B_{21} is the proportionality constant called Einstein coefficient for stimulated emission.

Under thermal equilibrium, the absorption rate or the number of photons absorbed per second is equal to the sum of the number of photons emitted per second by spontaneous and stimulated emissions.

Thus RHS of equation (1) is equal to sum of RHS of equations (2) and (3)

$B_{12}N_1\rho(\nu) = A_{21}N_2 + B_{21}N_2\rho(\nu)$ or $\rho(\nu)[B_{12}N_1 - B_{21}N_2] = A_{21}N_2$

$\rho(\nu) = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$ or $\rho(\nu) = \frac{A_{21}N_2}{B_{21}N_2\left(\frac{B_{12}N_1}{B_{21}N_2} - 1\right)}$

or $\rho(\nu) = \frac{A_{21}}{B_{21}\left(\frac{N_1B_{12}}{N_2B_{21}} - 1\right)}$.....(4)

According to Boltzmann, the atomic population at different energy levels at a given temperature T is given by (Boltzmann distribution law)

$N_1 = N_0e^{-(E_1/kT)}$ and $N_2 = N_0e^{-(E_2/kT)}$. where N_0 is the total number of atoms and k is the Boltzmann constant.

Dividing the above equations, $\frac{N_1}{N_2} = \frac{e^{-(E_1/kT)}}{e^{-(E_2/kT)}} = e^{(E_2-E_1/kT)}$

or $\frac{N_1}{N_2} = e^{(h\nu/kT)}$ ....(5) (since $E_2 - E_1 = h\nu$)

substituting for $\frac{N_1}{N_2}$ from (5) in (4) we get $\rho(\nu) = \frac{A_{21}}{B_{21}}\left[\frac{1}{\left(\frac{B_{12}}{B_{21}}\, e^{(h\nu/kT)} - 1\right)}\right]$.....(6)

The energy density of radiation at a given temperature as per Planck's radiation formula is

$\rho(\nu) = \frac{8\pi h\nu^3}{c^3}\left[\frac{1}{(e^{(h\nu/kT)} - 1)}\right]$.....(7)

Comparison of equations (6) and (7), we get $\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$.....(8)

and $\frac{B_{12}}{B_{21}} = 1.$ This implies $B_{12} = B_{21}$.....(9)

By substituting (8) in (7) $\rho(\nu) = \frac{A_{21}}{B_{21}}\left[\frac{1}{(e^{(h\nu/kT)} - 1)}\right]$

Rearranging, we have $\frac{\textit{Spontaneous emission probability}}{\textit{Stimulated emission probability}} = \frac{A_{21}}{B_{21}\rho(\nu)} = (e^{(h\nu/kT)} - 1)$ ......(10)

Case 1 : If $h\nu > kT$, $e^{(h\nu/kT)} \gg 1$ From eqn. (10), $\frac{A_{21}}{B_{21}} \gg 1$ then spontaneous emission probability is greater than stimulated emission probability which is the case in electronic transition in atoms and molecules.

Case 2 : If $h\nu \approx kT$, $e^{(h\nu/kT)}$ will be low and comparable to 1. Then A_{21} will be comparable to B_{21} i.e. the phenomena of stimulated emission becomes significant.

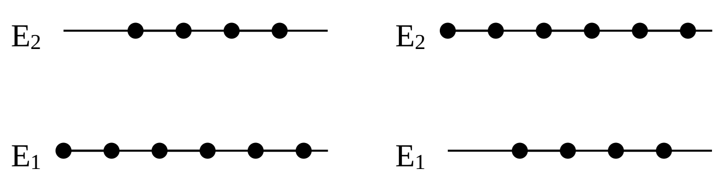
Case 3 : If $h\nu < kT$, $(e^{\left(\frac{h\nu}{kT}\right)} - 1) \ll 1.$ From eqn. (10), $\frac{A_{21}}{B_{21}} \ll 1,$ then stimulated emission probability is greater than spontaneous emission probability which is the case in atomic transitions in microwave regions and visible regions. This leads to laser action.

Population inversion

Consider a sample having a number of atoms in thermal equilibrium at a certain temperature. If the number N_1 of these atoms are in the ground state of energy E_1 and the number N_2 are in a state of higher energy E_2 then according to Boltzmann's law $\frac{N_2}{N_1} = exp\left[\frac{-(E_2 - E_1)}{kT}\right]$

Thus $N_2 < N_1$. There will always be less atoms in the excited state than in the ground state. The number of atoms present in a given energy state of a substance at thermal equilibrium is called **population** of that energy state.

The condition when the number of atoms in an excited state is more than that in the ground state is called **population inversion.**



The process of supplying energy from an external source, to achieve population inversion in a sample, is called **pumping.**

The process in which the atoms in a given sample are raised to higher energy states using light energy is called **optical pumping**. In this process, the sample is illuminated with light of frequency ν such that $h\nu = E_2 - E_1$. The atoms in the ground state absorb the energy of incident photons and jump to the higher energy state.
In an excited state, the atom remains for a short duration of time of around 10^{-8}s.

An excited state in which atoms can stay for a comparatively longer duration of time of around 5ms is called **metastable state.**

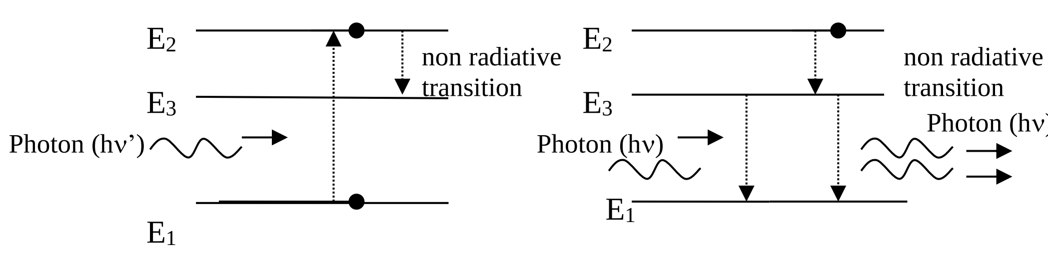
Components of laser : The components of a laser are
(i) active medium – collection of atoms/molecules or ions in a solid, liquid or a gas capable of amplifying light by way of population inversion.
(ii) the pumping source to achieve population inversion. This could be optical pumping or electric discharge or any other method to move the atoms to metastable state from ground state.
(iii) the optical resonator – a mechanism to retain the photons and make them move back and forth in the active medium with the help of mirrors to achieve amplification.

Principle of Laser

The action of laser is based on stimulated emission and amplification of light. In producing laser, the following conditions must be satisfied.
(1) State of population inversion,
(2) Existence of metastable state and
(3) confinement of emitted photons to achieve population inversion.

Laser action

Consider a system of atoms that exist in three different energy states namely, ground state(E_1), excited state (E_2) and metastable state (E_3) as shown in the diagram. In the excited state an atom can exist only for a time interval of 10^{-8} s. In the metastable state an atom can remain stable for a longer duration of time (≈5 ms).

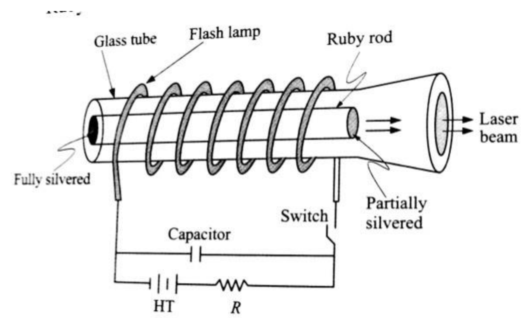


(a) The system of atoms in the ground state are illuminated by radiation such that they get excited and move to excited state (E_2) by absorbing photons of energy $h\upsilon' = E_2 - E_1$. This is called optical pumping.

(b) Since the life of excited atoms is very small, they jump to metastable state (E_3) by non radiative transition. The atoms can remain in this metastable state for longer time. Thus there will be more atoms in metastable state than in the ground state. Thus population inversion is said to be achieved.

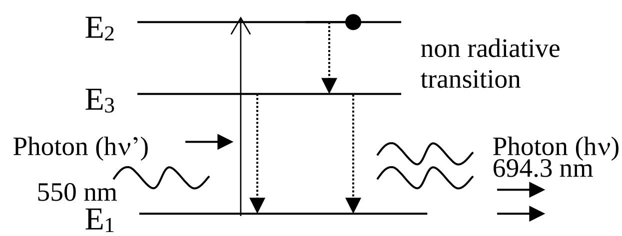
(c) The atoms in the metastable state are bombarded by photons each of energy $h\upsilon = E_3 - E_1$ The atoms make transition to the ground state by stimulated emission. This results in emission of photons each of energy $h\upsilon = E_3 - E_1$. These photons along with the bombarding photons have same energy and are in same phase. Thus the number of photons is multiplied by a factor of two. This process repeats and light amplification by stimulated emission of radiation or LASER occurs.

Ruby Laser



Construction : It is a three level solid state pulsed laser. It consists of a ruby rod which is a crystal of aluminum oxide with an addition of 0.01 to 0.05% chromium oxide. Chromium ions (Cr^{3+}) replace few aluminium atoms in the crystal lattice. The ends of the ruby rod are optically flat and perfectly parallel whose one end is fully silvered and the other end is partially silvered. The rod is inside a glass tube around which there is a helical xenon flash tube that acts as the optical pumping system.

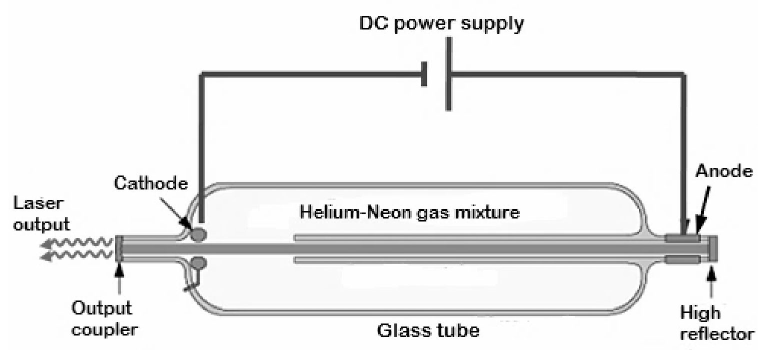
Working:



1. Chromium ions are excited from level E_1 to E_2 by the absorption of light of wavelength *550 nm* from the xenon flash tube. These ions quickly undergo transition to energy level E_3 called metastable state by radiating heat. The life time of this state is around 3 ms. Thus the population of the E_3 state becomes more than that of E_1 state. This results in population inversion.
2. There will be some photons produced by spontaneous transition from E_3 to E_1 and have a wavelength of 694.3 nm. These photons are reflected back and forth at the silvered surfaces of the ruby rod.

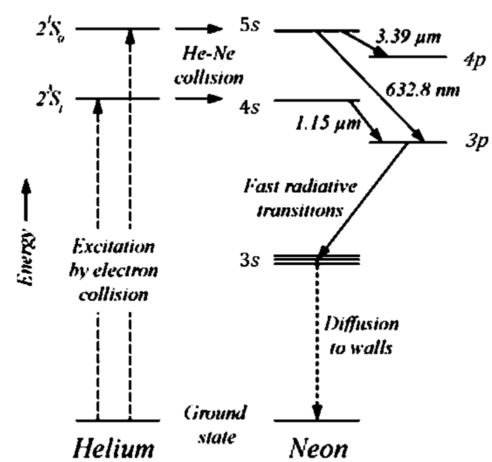
3. The photons that are moving parallel to the axis of the rod resonate and stimulate the emission of similar other photons. The emitted photons multiply resulting in a beam of photons that are moving parallel to the rod which are coherent and monochromatic. When the beam develops sufficient intensity, it emerges from the partially silvered end of the rod as Laser light.

Helium Neon Laser



Construction : A helium–neon laser or **He-Ne laser**, is a four level gas laser whose medium consists of a mixture of helium and neon (in the ratio 10:1) inside small bore capillary tube, excited by a DC electrical discharge. The He-Ne laser operates at a wavelength of 632.8 nm, in the red part of the visible spectrum having a average power output of 50 mW. In He-Ne laser, neon atoms are the active centers and have energy levels suitable for laser transitions while helium atoms help in exciting neon atoms.

Electrodes (anode and cathode) are provided in the glass tube to send the electric current through the gas mixture. These electrodes are connected to a DC power supply. The glass tube (containing a mixture of helium and neon gas) is placed between two parallel mirrors with the right mirror completely silvered and the left one partially silvered through which laser light comes out.



Working :

1. When the power is switched on, a high voltage of about 10 kV is applied across the gas mixture. The electrons produced in the process of discharge are accelerated between the electrodes through the gas mixture which transfer some of their energy to the helium atoms in the gas. As a result, **electrons of the helium atoms jumps to their 2^3S_1 and 2^1S_0 excited metastable states** by absorbing energies of 19.78eV and 20.62eV energy respectively. These energies are equal to the energy required to excite the Neon atoms to 4s and 5s states.
2. When the excited electrons of the helium atoms collide with the lower energy state electrons of the neon atoms, they transfer energy to the neon atoms. As a result, the **lower energy state electrons of the neon atoms jumps into their excited metastable states** as energy levels of $5s_2$ and $4s_2$ excited levels of Neon atoms are

identical to that of metastable states of the helium atoms. Thus, **helium atoms help neon atoms in achieving population inversion**.

3. After some period, the metastable state electrons of the neon atoms will spontaneously fall into the next lower energy states by releasing photons. The Ne atom makes three laser transitions as shown in the energy level diagram. The only transition that results in light in the visible region is **5s to 3p transition resulting in light of wavelength 632.8 nm.**
4. The photons emitted from the neon atoms will moves back and forth between two mirrors until it stimulates other excited electrons of the neon atoms and causes them to **emit laser light of wavelength 632.8 nm**.

Differences between Ruby laser and He – Ne laser

	Ruby laser	He –Ne laser
1	It is solid state laser	Gas laser
2	It is a three level energy system	It is four level energy system
3	It is a pulsed laser	It is a continuous laser
4	Optical pumping is used for population inversion	Electric discharge is used for population inversion
5	Large heat is generated. Thus cooling mechanism is required.	It does not get heated and can work continuously. No cooling system required.
6	Optical pumping is difficult as more than half the atoms need to be excited from ground state.	Due to the four level system is easier to excite the atoms
7	High intense flash light is required for pumping.	Low power of 5W to 10W is sufficient for excitation.
8	Output power of laser is spikes of 10^4 to 10^5 W	Output power is continuous laser of 0.5 mW to 50mW.
9	Wavelength of laser is 694.3nm	Wavelength of laser is 632.8 nm

Properties of Laser

1. Laser light is highly monochromatic
2. Laser light is highly coherent
3. Laser light is highly directional
4. Laser light has high intensity
5. Laser light can be sharply focused

Monochromaticity

(a) Laser light consists of essentially one wavelength, having its origin in stimulated emission from one set of atomic energy levels. This is possible because laser transition involves well-defined energy levels.

(b) Electromagnetic wave of frequency $\nu = \frac{(E_2 - E_1)}{h}$ only can be amplified, $\Delta\nu$ has a certain range which is called line width (range of frequencies or wavelengths) or bandwidth. This line width is decided by various broadening factors such as Doppler effect of moving atoms and molecules.

(c) The generation of laser is such that the laser cavity forms a resonant system and laser oscillation is sustained only at the resonant frequencies of the cavity. This leads to further narrowing of the laser line width. So laser light is usually very pure in wavelength, having property of narrow linewidth and high monochromaticity. For a He-Ne laser of wavelength of 632.8nm the wavelength bandwidth is about 0.01nm

Coherence : During the stimulated emission, a passing excited photon stimulates a transition from a higher level to the lower level, thus resulting in the emission of two photons, The two emitted photons are said to be in phase, which means that the crest or the trough of the wave associated with one photon will occur at the same time as on the wave associated with the other photon. An avalanche of similar photons is created and these photons have a fixed phase relationship with each other. They are said to possess the property of coherence. This fixed phase relationship between the photons results in laser beam.

A wave which appears to be a pure sine wave for an infinitely large period of time or in an infinity extended space is said to be a perfectly coherent wave. In such a wave there is a definite relationship between 1. phase of wave and a given time and at a certain time later (Temporal) and 2. Phase of a wave at a point and at a certain distance away (Spatial).

There are two types of coherence –**spatial coherence** and **temporal coherence**

Spatial Coherence : A beam of light is said to be spatially coherent, if the phase difference of the waves crossing two points lying on a plane perpendicular to the direction of propagation of the beam is independent of time. It is the measure of Correlation between waves at different points in space.

To understand coherence, let us take two points on a wave front, at time equal to zero. There will be a certain phase difference between these two points and if it remains same even after lapse of a period of time, then the electromagnetic wave (em) has perfect coherence between the two points. In case, the phase difference remains same for any two points anywhere on the wave front, then we say that the electromagnetic wave has perfect **spatial coherence**, where as if this is true only for a specific area, then the electromagnetic wave is said to have only **partial spatial coherence**. Spatial coherence is related to directionality and uniphase wave fronts.

Temporal Coherence : Temporal coherence is the measure of the average correlation between the value of a wave and itself delayed by a time τ, at any pair of times. Temporal coherence tells us how monochromatic a source is. In other words, it characterizes how well a wave can interfere with itself at a different time.

Coherent time τ_c : The time during which the phase or amplitude of a wave remains constant before it undergoes change is called coherent time. At a delay of τ=0 the degree of coherence is perfect, whereas it drops significantly as the delay passes $\tau = \tau_c$.

The **coherence length L_c** is defined as the distance the wave travels in time τ_c. It is the distance travelled by the wave during which the phase or the amplitude of the wave remains constant. It is given by $L_c = c\,\tau_c$ where c is the speed of light.

In radio band systems, the coherent length is approximates $L_C = \frac{c}{n\,\Delta f} = \frac{\lambda^2}{n\,\Delta\lambda}$ where n is the refractive index of the medium and Δf is the bandwidth. For a laser L_C is very large.

The coherence length can be measured using a Michelson interferometer and is the optical path length difference of a self-interfering laser beam which corresponds to a 1/e = 37% fringe visibility, where the fringe visibility is defined as $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$ where I is the fringe intensity.

Note : 1. A wave containing only a single frequency (monochromatic) is perfectly correlated with itself at all time delays.

2. Conversely, a wave whose phase drifts quickly will have a short coherence time.

3. Pulses (wave packets) of waves, which naturally have a broad range of frequencies, also have a short coherence time since the amplitude of the wave changes quickly.

4. White light, which has a very broad range of frequencies, is a wave which varies quickly in both amplitude and phase. Since it consequently has a very short coherence time (just 10 periods or so), it is often called incoherent.

5. Monochromatic sources are usually lasers; such high monochromaticity implies long coherence lengths (up to hundreds of meters). For example, a stabilized and mono mode helium–neon laser can easily produce light with coherence lengths of 300 m. LEDs are characterized by Δλ ≈ 50 nm, and tungsten filament lights exhibit Δλ ≈ 600 nm, so these sources have shorter coherence times than the most monochromatic lasers.

An 800 nm laser diode with a 1 nm spectral width would have a coherence length of about 0.64 mm. A 600 nm LED with a spectral width of 60 nm would have a coherence length of around 6 μm.

Directionality : One of the important properties of laser is its high directionality. The mirrors placed at opposite ends of a laser cavity enables the beam to travel back and forth in order to gain intensity by the stimulated emission of more photons at the same wavelength, which results in increased amplification due to the longer path length through the medium. The multiple reflections also produce a well-collimated beam, because only photons traveling parallel to the cavity walls will be reflected from both mirrors. If the light is the slightest bit off axis, it will be lost from the beam. The resonant cavity, thus, makes certain that only electromagnetic waves traveling along the optic axis can be sustained, consequent building of the gain.

Beam divergence : The oscillation of the beam in the resonator cavity produces a narrow beam that subsequently diverges at some angle depending on the resonator design, the size of the output aperture, and resulting diffraction effects on the beam. These diffraction effects usually referred as a beam-spreading effect are a result of the light waves passing through a small opening. These diffraction phenomena impose a limit on the minimum diameter of a light point after passing through an optical system. For a laser, the beam emerging from the output mirror can be thought of as the opening or aperture, and the diffraction effects on the beam by the mirror will limit the minimum divergence and spot size of the beam.

The divergence angle describes the directionality of the laser. For a perfect spatially coherent laser beam, the diffraction limited divergence angle θ is given by $d\theta = \frac{1.22\lambda}{d}$ where d is the diameter of the front mirror. In case of gas lasers $d\theta$ is as small as 10^{-5} to 10^{-6} radian.

The optical intensity I, of a laser beam, is the optical power per unit area, which is transmitted through an imagined surface perpendicular to the propagation direction. The units of the optical intensity (or light intensity) are W/m^2 or (more commonly) W/cm^2.

It's the coherent, organized property of laser light that makes it capable of delivering a high amount of energy in a small beam. In the case of visible lasers, this makes the laser beam very bright and intense. Due to the laser light's parallelism, it can be focused very efficiently compared with other types of light.

Estimation of Intensity and energy density :

For a plane electromagnetic wave generated by a continuous wave laser, the intensity (power per unit area) has a magnitude: $I = \frac{E\,B}{\mu_0}$, But $B = \frac{E}{c}$,

The magnitude of instantaneous intensity is $I = \frac{E^2}{c\,\mu_0} = \frac{cB^2}{\mu_0}$

The average intensity is $I_{av} = \frac{E_{rms}\,B_{rms}}{\mu_0} = \frac{E_{max}\,B_{max}}{2\mu_0} = \frac{E_{max}^2}{2c\mu_0} = \frac{c\,B_{max}^2}{2\mu_0}$

The energy density, in units of J/m^3, is: $u = u_E + u_B = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\,\mu_0} = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$

Exactly half the energy in an EM wave is in the electric field, and half in the magnetic field.

Applications of Laser

1. Laser is used in laser Raman spectroscopy to understand the molecular structure.
2. The distance between two objects can be found accurately using laser reflectors.
3. They are used for cutting, drilling and welding of hard materials.
4. Lasers are used optical communication systems.
5. Lasers are extensively used in holography and its applications.
6. Laser is used in the measurement of pollutants in the atmosphere and water.
7. Lasers are used to read compact discs.
8. In surgery, control of bleeding is achieved with the use of laser where minimally invasive surgical techniques are used. Lasers are used in bloodless surgery to correct eye disorders, to cut or destroy tissues that are abnormal, to shrink or destroy tumors or lesions etc…
9. **Holography** is the science of making **holograms**. Typically, a hologram is a photographic recording of a light field, rather than of an image formed by a lens, and it is used to display a fully three-dimensional image of the holographed subject, which is seen without the aid of special glasses. In its pure form, holography requires the use of laser light for illuminating the subject and for viewing the finished hologram. In laser holography, the hologram is recorded using a source of laser light, which is very pure in its color and orderly in its composition. All holograms involve the interaction of light coming from different directions and producing a microscopically fine interference pattern which a plate, film, or other medium photographically records.

Descriptive questions

1. (a) Explain stimulated absorption, spontaneous and stimulated emission of radiation with the help of energy level diagram.
   (b) What is the principle of working of a laser?

2 (a) What is stimulated emission of radiation? Explain.
   (b) Derive the relation between transition probabilities of spontaneous and stimulated emissions in terms of Einstein coefficients.

3. (a) What is population inversion? Explain.
   (b) Explain the working of a laser.

4 (a) Mention four characteristic properties of laser light.
   (b) Describe the construction and working of He – Ne laser. Also draw the energy level diagram.

5 (a) Describe the construction and working of a Ruby laser. Explain the energy level diagram
   (b) Mention any two applications of Laser light.

6 (a) What are spatial and temporal coherence? Explain.
(b) What are coherence time and coherence length? Explain.
(c) Write a note on directionality and divergence of laser.

Numerical problems

1 A laser beam with power per pulse 2 mW lasts for 10 ns, contains 7.5×10^7 photons per pulse. Calculate the wavelength of laser light.
[$P = \frac{E}{t}$ *and* $E = \frac{n\,h\,c}{\lambda}$ λ = 745 nm]

2 Calculate the energy difference in eV between two levels of a gas laser, the transition between which results in the emission of light of wavelength 634.8 nm. Also calculate the number of photons emitted per second if the optical power output is 1.2 mW.
[$\Delta E = \frac{hc}{\lambda}$, $n = \frac{P}{\Delta E}$. ΔE = 1.95 eV, n = 3.87×10^{15}]

3 The ratio of population in upper to lower energy states at room temperature of 300K is 1/e. Determine the wavelength of radiation emitted at room temperature, given $h = 6.625 \times 10^{-34}\,Js$, $c = 3 \times 10^8\,ms^{-1}$, $k = 1.38 \times 10^{-23} JK^{-1}$.
[$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/kT}$, *and* $\frac{N_2}{N_1} = \frac{1}{e}$, *Thus* $\frac{1}{e} = \frac{1}{e^{(E_2 - E_1)/kT}}$ *or* $1 = \frac{E_2 - E_1}{kT}$ *or* $1 = \frac{hc}{kT\lambda}$, $\lambda = 53.8\,\mu m.$

4 At thermal equilibrium, the ratio of the number of spontaneous to stimulated emissions is found to be 10^{10} at a frequency of 5×10^{14}Hz. Find the ratio of the number of atoms in the higher level to the lower level in a 2 – level system at equilibrium.
[$B_{12}N_1\rho(\nu) = A_{21}N_2 + B_{21}N_2\rho(\nu)$ and $B_{12} = B_{21}$. Thus $B_{21}N_1\rho(\nu) = A_{21}N_2 + B_{21}N_2\rho(\nu)$
$N_2 = \frac{B_{12}N_1\rho(\nu)}{A_{21} + B_{21}\rho(\nu)}$ Thus $\frac{N_2}{N_1} = \frac{B_{12}\rho(\nu)}{A_{21} + B_{21}\rho(\nu)}$, $\frac{N_2}{N_1} = \frac{1}{\frac{A_{21}}{B_{12}\rho(\nu)} + 1} = \frac{1}{10^{10}+1} = 10^{-10}$]

5 Calculate the ratio of stimulated to the spontaneous emission rate at a temperature of 250⁰C for sodium D line of 590 nm.
[As $\frac{A_{21}}{B_{21}\rho(\nu)} = \left(e^{(h\nu/kT)} - 1\right)$ $R = \frac{\boldsymbol{B_{21}}}{\boldsymbol{A_{21}}} = \frac{\boldsymbol{1}}{\boldsymbol{e^{(h\nu/kT)} - 1}} = \frac{\boldsymbol{1}}{\boldsymbol{e^{(hc\lambda/kT)} - 1}} = \frac{\boldsymbol{1}}{\boldsymbol{E^{46.7} - 1}} = 5.23 \times 10^{-21}$ **]**

6 At what temperature are the rates of spontaneous and stimulated emission are equal ? Given : $\lambda = 500$ nm.
[$\frac{A_{21}}{B_{21}\rho(\nu)} = 1 = \left(e^{(h\nu/kT)} - 1\right)$ *or* $e^{(T)} = 2$ *or* $\frac{hc}{\lambda kT} = log_e 2$ T = 46,633 K]

7 A laser source emits wavelength of 632.8 nm and has an output power of 5 mW. How many photons are emitted per second by this laser?
[$E = \frac{hc}{\lambda} = 3.148 \times 10^{-19} J$, $n = \frac{P}{E} = 1.59 \times 10^{16}$]

8 In a He – Ne laser, transition from 5S – 3P level gives a laser emission of wavelength 632.8 nm. If the 3P level has energy 15.2×10^{-19}J, calculate the pumping energy required, assuming no loss.

[$E = \frac{hc}{\lambda} = 3.148 \times 10^{-19} J, Total\ energy\ required = 15.2 \times 10^{-19} + 3.15 \times 10^{-19} = 18.35 \times 10^{-19} J$

9 A pulse from a laser with 1mW power lasts for 10 ns. If the number of photons emitted per second is 3.491×10^7, calculate the wavelength of laser.

[$E = \frac{P}{n} = \frac{1\times10^{-3}\times10\times10^{-9}}{3.491\times10^{7}} = 2.864 \times 10^{-19}$, $\lambda = \frac{hc}{E} = 694.26\ nm$]

10 In a Ruby laser, the total number of Cr ions is 2.8×10^{19}. If the laser emits radiation of wavelength 700 nm, calculate the energy of the laser pulse in eV.

[$E = \frac{hc}{\lambda\times1.6\times10^{-19}} = 1.77\ eV, Total\ energy\ E_T = E \times n = 4.95 \times 10^{19} eV$]

11 Estimate the order of magnitude of the standing waves in a laser when the length of resonating cavity is 1 m and wavelength is 632.8 nm.

[Length of resonating cavity = integral multiples of $\lambda/2$. L = $\frac{n\lambda}{2}$, $n = 3.16 \times 10^6$]

12 Determine the intensity of a laser beam of power 50 mW having a diameter of 1.5 mm. Assume the intensity across the beam is uniform.

[$I = \frac{P}{A} = \frac{P}{\pi r^2} = \frac{4P}{\pi d^2} = 2.8 \times 10^4\ Wm^{-2}$]

13 A ruby laser emits light of wavelength 694 nm. The duration of pulses is 0.1 ns. Calculate the coherent length, bandwidth and linewidth.

[$L_C = c\,\tau_c = 0.03\ m$, $Bandwidth\ \Delta\nu = \frac{1}{\tau_c} = 10^{10} Hz$, $Linewidth = \Delta\lambda = \frac{\lambda^2}{c}\Delta\nu = 0.016\ nm$]

14 A laser beam of wavelength 694.3 nm is focused on a surface area of 0.25 mm diameter. If the power of the laser source is 10 mW, Calculate the intensity and energy of the emitted photons.

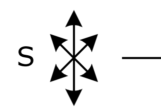
[$I = \frac{P}{A} = \frac{4P}{\pi d^2} = 2.05 \times 10^5\ Wm^{-2}$, $E = \frac{hc}{\lambda} = 2.86 \times 10^{-19}\ J$]

Syllabus : Paper 401 – Unit III : Polarization - Review of plane polarized light and method of production; Double refraction at crystals; Huygens' explanation of double refraction; Theory of retarding plates - Quarter wave plates and Half wave plates; Theory of superposition of two plane polarized waves with perpendicular vibrations, Production and detection of linearly , elliptically and circularly polarized light; Optical activity - Fresnel's explanation, Laurent's half shade polarimeter

Introduction : The phenomenon of restricting the vibration of a light wave to a particular direction in a plane perpendicular to the direction of propagation of light is called **polarization of light.**

According to Maxwell's electromagnetic theory, light waves are transverse in nature with the electric and magnetic field vectors vibrating at right angles to each other and both are perpendicular to the direction of propagation of light.

Light is represented by the vibrations of electric field vector ($\vec{E}$). In an ordinary light (unpolarized light), vibrations of electric field vector are in every plane perpendicular to direction of propagation of light as shown in the diagram.

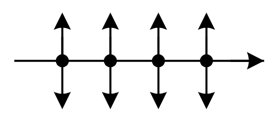


Unpolarised light source (right angles to plane of paper)

Pictorial representation

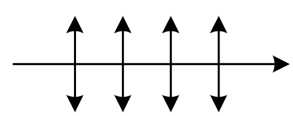
(1) Unpolarized or ordinary light:

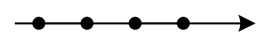
The propagation of light is represented by electric field vectors vibrating in the vertical direction in the plane of the paper (arrows) and in the horizontal direction perpendicular to plane of paper (dots).


Unpolarised light

(2) Plane Polarized light:

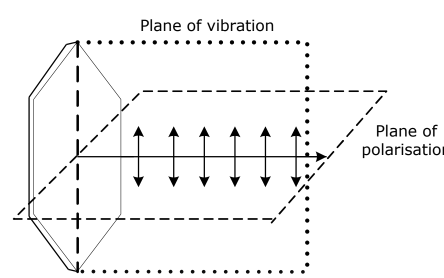
The propagation of polarized light is represented by electric field vectors vibrating in the vertical direction in the plane of the paper (arrows) or by electric field vectors vibrating in the horizontal direction perpendicular to plane of paper (dots). Here the vibrations are restricted to only one plane.


Plane polarised light (vibrations parallel to plane of paper)


Plane polarised light (vibrations perpendicular to plane of paper)

Plane of polarisation and plane of vibration

In a plane polarized light, the plane containing the direction of vibration and direction of propagation of light is called **plane of vibration.**

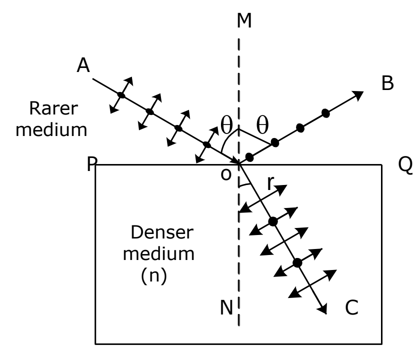


In a plane polarized light, the plane that does not contain the direction of vibration and plane which is perpendicular to plane of vibration is called **plane of polarization.** It also contains the direction of propagation of light.

Methods of producing plane polarised light

The methods of producing plane polarised or linearly polarised light are
(1) Reflection, (2) Refraction, (3) Double refraction and (4) Selective absorption and (5) Scattering.

1. Polarisation by reflection:



Mallus discovered this method in 1808. He found that light reflected from the surface of glass was either partially polarised or completely polarized and the degree of polarization depends on the angle of incidence. As the angle of incidence is increased the degree of polarisation gradually increases. The reflected light is completely polarised for a particular angle of incidence called **polarizing or Brewster's angle (θ_p).**

Polarising angle for a given pair of media is that angle of incidence at which the reflected light is completely plane polarised.

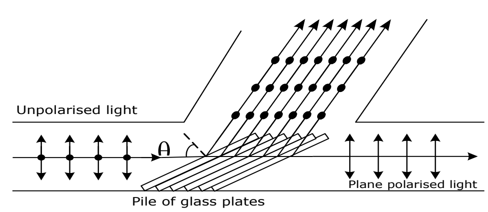
Brewster analyzed the property of polarization by reflection and the conclusions are
1. For a particular angle incidence called polarizing angle, the reflected light is completely plane polarized.
2. The refractive index of the medium is equal to the tangent of the polarising angle for that medium. This called the **Brewster's law.**
3. Mathematically, Brewster's Law is given by $\boldsymbol{n = tan\theta_P}$ ....(1)

Fro (1), $n = \frac{\sin\theta_P}{\cos\theta_P}$ ....(2) From Snell's law $n = \frac{\sin i}{\sin r} = \frac{sin\theta_P}{\sin r}$ ...(3)

From (2) and (3) $\frac{sin\theta_P}{\sin r} = \frac{\sin\theta_P}{\cos\theta_P}$ or $\sin r = \cos\theta_P = \sin(90 - \theta_P)$ or $r = 90 - \theta_P$

Thus $\boldsymbol{r + \theta_P = 90}$. At the polarizing angle of incidence, the reflected and refracted rays of light are perpendicular to each other.

2. Polarisation by refraction:



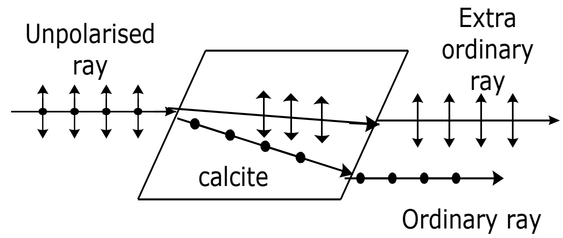
A ray of unpolarized light is incident on a number of parallel glass plates, with the angle of incidence is equal to the polarising angle θ_p.

The reflected light is completely polarised having vibrations lying in a plane perpendicular to plane of incidence (dots). The refracted light, after a series of reflections and refractions will have only vibrations lying in the plane parallel to the plane of incidence (arrows) and the emergent beam is said to be plane polarized.

3. Polarisation by selective absorption – Dichroism : The property of a doubly refracting crystal to absorb one of the refracted rays and allow the other refracted ray to emerge from it is called **selective absorption or dicroism**. Crystals exhibiting this property are called dichroic crystals. Tourmaline is a crystal having special property of absorbing the ordinary ray and the extra ordinary ray to different extents.

4. Polarisation by double refraction:

The phenomenon exhibited by certain crystals in which an incident beam is split into two refracted beams is called **double refraction**. This property was discovered by **Esmus Bartholinus**. Crystals like calcite, quartz, mica etc. exhibit this property. The crystals having this property are said to exhibit **birefringence.**



When a calcite crystal is placed on a paper having ink mark, two images of ink mark are seen when viewed through the crystal from above. If the crystal is rotated about the direction of incident beam, one of the images remains stationary (ordinary image) while the other revolves round the first image (extraordinary image).

The refracted ray that obeys the laws of refraction is called **ordinary ray (O –Ray).**

The refracted ray that does not obey the laws of refraction is called **extraordinary ray (E - O –Ray).** Both the refracted rays are plane polarized and their planes of vibration mutually perpendicular to each other.

Ordinary Ray (O – Ray)	Extra ordinary ray (E – O - Ray)
Obeys the laws of refraction	Does not obey the laws of refraction
Refractive index remains constant for any angle of incidence.	Refractive index varies with the angle of incidence.
The O-ray travels with the same speed in all directions.	The E – O Ray travels with different speeds in different directions.
They are plane polarized	They are plane polarized.
O-ray is polarised in the plane of principal section.	The E-O-ray is polarised in a plane perpendicular to the principal section.
The O-ray and the E-O-ray travel with the same speed along a particular direction inside the crystal and this direction is called **optic axis**. Along optic axis double refraction is not observed.	

Uniaxial crystal is a crystal with one optic axis. There are two types of uniaxial crystals. They are negative crystals and positive crystals. For E.g. Calcite (negative), quartz (positive), tourmaline etc… **Biaxial crystal** has two optic axes. E.g. Selenite, mica etc…
Along optic axis $\boldsymbol{v_o = v_e}$**.** (No double refraction) Along a direction other than optic axis, the velocity of ordinary ray v_o is not equal to velocity of extraordinary ray v_e.

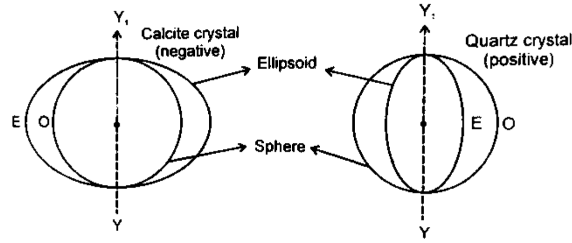
Negative crystals	Positive crystals
Crystal in which $\boldsymbol{v_o < v_e}$ are called **negative crystals**.Eg. Calcite, mica,--	Crystal in which $\boldsymbol{v_o > v_e}$ are called **positive crystals**. Eg. Quartz, topaz….
Velocity of ordinary ray is constant in all directions.	Velocity of ordinary ray is constant in all directions.
Velocity of E – O ray is different in different directions. It is equal to that of O ray along optic axis and is **minimum**. It is **maximum** along right angles to optic axis.	Velocity of E – O ray is different in different directions. It is equal to that of O ray along optic axis and is **maximum**. It is **minimum** along right angles to optic axis.
The refractive index for ordinary ray is greater than that for E – O ray ordinary ray $\boldsymbol{n_o > n_e}$**.** Along the optic axis $n_o = n_e$. The refractive index for E-O ray is called **principal refractive index** $\boldsymbol{n_E}$ when the ray is travelling perpendicular to optic axis and its value is minimum as the velocity is maximum.	The refractive index for ordinary ray is less than that for E – O ray ordinary ray $\boldsymbol{n_o < n_e}$**.** Along the optic axis $n_o = n_e$. The refractive index for E-O ray is called **principal refractive index** $\boldsymbol{n_E}$ when the ray is travelling perpendicular to optic axis and its value is maximum as the velocity is minimum.
The wavefront of O ray (spherical) lies **inside** that of E O ray (ellipsoid).	The wavefront of O ray (spherical) lies **outside** that of E O ray (ellipsoid).

Note : A plane which contains the optic axis and perpendicular to the opposite faces of a crystal is called the **principal section of the crystal**. O-ray is polarised in the plane of principal section and the E-O-ray is polarised in a plane perpendicular to the principal section.

Huygens' theory of double refraction

1. According to Huygen each point on a wavefront act as a fresh source of disturbance and sends out secondary wavelets
2. When such a wavefront strikes a doubly refracting crystal, every point of the crystal becomes the source of two wavefronts. They are (a) Ordinary wavefront corresponding to ordinary rays. They travel with the same speed in all directions. The secondary

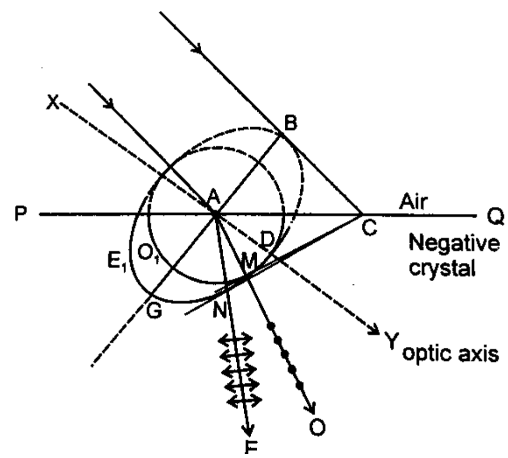
wavefront is spherical. (b) Extra-ordinary wavefront corresponding to extra-ordinary rays. They travel with different speeds in different directions. The secondary wavefront is ellipsoidal.



3. The sphere and ellipsoid touch each other at points which lie on the optic axis (YY) of the crystal. This is because, both O – ray and E-O-ray travel with the same speed along the optic axis.
4. In negative crystals like calcite, the ellipsoid lies outside the sphere. This shows that the extra-ordinary wavefront travel faster than the ordinary wavefront as shown except along the optic axis.
5. In positive crystals like quartz, the ellipsoid lies inside the sphere. This shows that the extra-ordinary wavefront travel slower than the ordinary wavefront as shown except along the optic axis.

Huygens' theory of double refraction can be applied to uniaxial crystals as explained below.

Optic axis in the plane of incidence and inclined to the refracting surface



Oblique incidence : Let AB be the plane wavefront incident obliquely on the surface PQ of a negative crystal. The optic axis XY is in the plane of incidence and inclined to refracting surface as shown.

When the wavefront AB touches the surface of the PQ of the crystal, Point A becomes the source of two wavefronts, namely O_1 ordinary spherical wavefront for the ordinary rays and E_1 ellipsoidal wavefront for the extra-ordinary rays. CM is the tangent to the ordinary wavefront O_1 called the refracted wavefront for O – ray. CN is the tangent to extra-ordinary wavefront E_1 called refracted wavefront for the E – ray as shown.

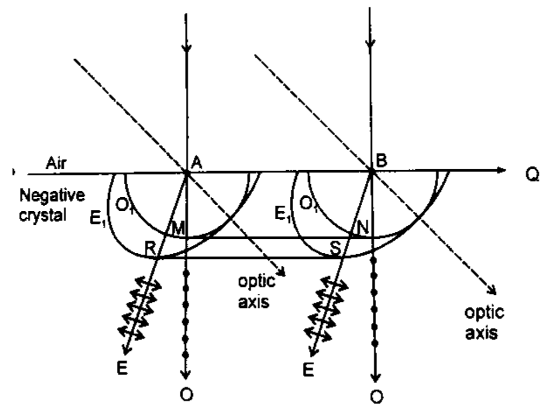
If v is the velocity of light in air, then time taken by light to reach C from B is given by $t = \frac{BC}{v}$. During this time ordinary ray travelling with velocity v_o will travel from A to M and extra-ordinary ray travelling with velocity v_e will travel from A to N. Therefore

$\frac{BC}{v} = \frac{AM}{v_o} = \frac{AN}{v_e}$ Thus $AM = \frac{BC}{v} v_o = \frac{BC}{n_o}$ (since $n_o = \frac{v}{v_o}$)

and $AN = \frac{BC}{v} v_e = \frac{BC}{n_e}$ (since $n_e = \frac{v}{v_e}$)

where n_o and n_e are the refractive indices of the medium for ordinary and extra-ordinary rays. The ordinary and extra-ordinary rays travel with different velocities along different directions. Here the semi major axis of the ellipsoid is $a = \frac{BC}{n_E}$ where n_E is the principal refractive index of the medium for the extra-ordinary ray. Here $n_E < n_e < n_o$.

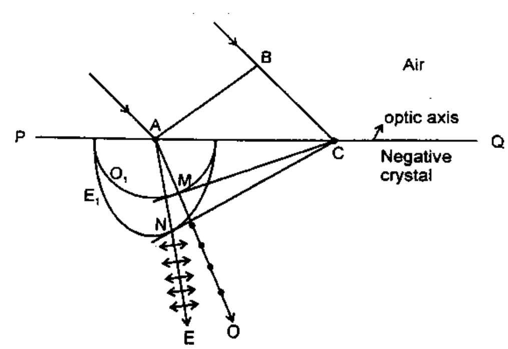
Normal incidence : Let AB be the plane wavefront incident normally on the surface PQ of a negative crystal. The optic axis XY is in the plane of incidence as shown.



When the wavefront AB touches the surface of the PQ of the crystal, Point A becomes the source of two wavefronts, namely O_1 ordinary spherical wavefront for the ordinary rays and E_1 ellipsoidal wavefront for the extra-ordinary rays. Both ordinary and extra-ordinary wavefronts MN and RS are parallel to the refracting surface. Since AO is perpendicular to MN, AE is not perpendicular to RS, the ordinary and extra-ordinary rays travel with different velocities as shown.

Optic axis in the plane of incidence and parallel to the refracting surface

Oblique incidence : Let AB be the plane wavefront incident obliquely on the surface PQ of a negative crystal. The optic axis XY is in the plane of incidence and parallel to refracting surface as shown.



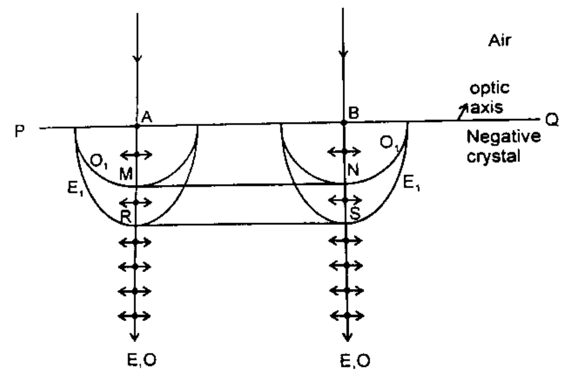
When the wavefront AB touches the surface of the PQ of the crystal, Point A becomes the source of two wavefronts, namely O_1 ordinary spherical wavefront for the ordinary rays and E_1 ellipsoidal wavefront for the extra-ordinary rays. The two wavefronts touch each other along the optic axis i.e. PQ. CM is the tangent to the ordinary wavefront O_1 called the refracted wavefront for O – ray. CN is the tangent to extra-ordinary wavefront E_1 called refracted wavefront for the E – ray as shown.

If v is the velocity of light in air, then time taken by light to reach C from B is given by $t = \frac{BC}{v}$. During this time ordinary ray travelling with velocity v_o will travel from A to M and extra-ordinary ray travelling with velocity v_e will travel from A to N. Therefore

$$\frac{BC}{v} = \frac{AM}{v_o} = \frac{AN}{v_e} \qquad \text{Thus} \qquad AM = \frac{BC}{v} v_o = \frac{BC}{n_o} \quad (\text{since } n_o = \frac{v}{v_o})$$

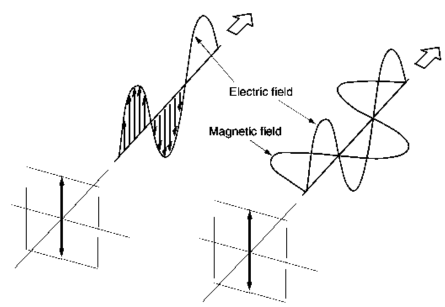
and $AN = \frac{BC}{v} v_e = \frac{BC}{n_e}$ (since $n_e = \frac{v}{v_e}$)

where n_o and n_e are the refractive indices of the medium for ordinary and extra-ordinary rays. The ordinary and extra-ordinary rays travel with different velocities along different directions. Here the semi major axis of the ellipsoid is $a = \frac{BC}{n_E}$ where n_E is the principal refractive index of the medium for the extra-ordinary ray. Here $n_E < n_e < n_o$.



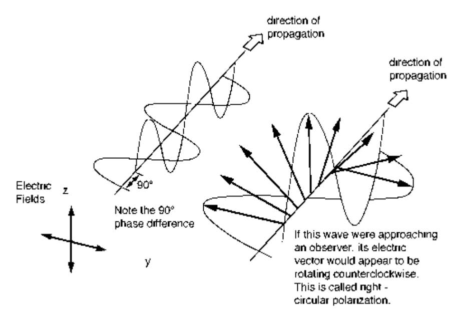
Normal incidence : Let AB be the plane wavefront incident normally on the surface PQ of a negative crystal. The optic axis XY is in the plane of incidence as shown.

When the wavefront AB touches the surface of the PQ of the crystal, Point A becomes the source of two wavefronts, namely O_1 ordinary spherical wavefront for the ordinary rays and E_1 ellipsoidal wavefront for the extra-ordinary rays. Both ordinary and extra-ordinary wavefronts MN and RS are parallel to the refracting surface. Since AO is perpendicular to MN, AE is also perpendicular to RS, the ordinary and extra-ordinary rays travel with different velocities in the same direction as shown. There is a definite phase difference introduced between the two rays which is the principle used in quarter wave plate and half wave plate.

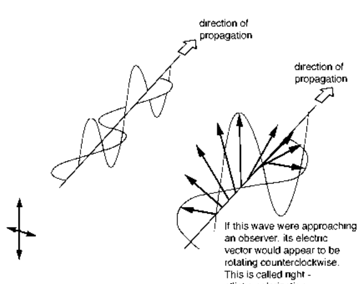


Types of Polarised light

Plane polarized light : If the vibrations of the electric vector of light in the medium are in a plane or linear and perpendicular to direction of wave propagation, then it is called linearly or plane polarized light. It is represented in the diagram as shown.

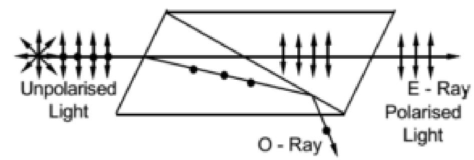


Circularly polarized light : If light waves are composed of two plane waves of equal amplitude having a phase difference of 90°, then the light is said to be circularly polarized. The electric vibrations are of equal amplitude with their tip executing a circle with constant period as shown.



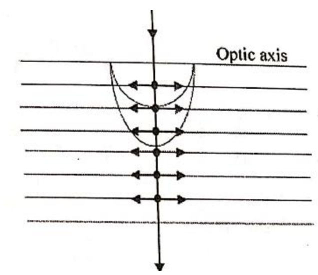
Elliptically polarized light : If two plane waves of different amplitude having a phase difference of 90° or if the relative phase is other than 90° then the light is said to be elliptically polarized. Here the electric vibrations are of different amplitudes and their tip executes an elliptical path of constant period. Circularly polarized light and plane polarized light are special cases of elliptically polarized light.

Nicol prism is an optical device used to produce plane polarised light and to analyze light. It was designed by William Nicol in 1828. It works on the principle of double refraction. It is made of calcite crystal with its length three times its width. It is cut diagonally into two parts and joined by a cement called Canada balsam.



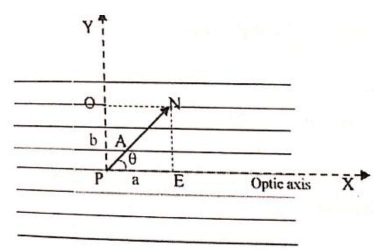
When beam of unpolarised light is incident on a Nicol prism, double refraction occurs with O-ray and E-O-rays plane polarized perpendicular to each other. The refractive index of the medium for the O – ray is 1.65 and that for E – O ray is 1.48 and the refractive index of Canada balsam (n_c) is 1.55. The O-ray is eliminated by total internal reflection (as $n_o > n_c$]. Only E-O-ray which is plane polarized emerges out of the Nicol prism (as $n_e < n_c$) as shown.

Theory of Retarding plates



Unpolarised monochromatic light gets plane polarized when it passes through a Nicol prism. This polarised light is allowed to fall normally on a doubly refracting uniaxial plate (made of calcite crystal) with its surfaces cut parallel to its optic axis.

The incident light is split into ordinary and extra-ordinary rays. The E ray has its vibrations parallel to the optic axis and O ray has its vibrations perpendicular to the optic axis both travelling in the same direction. The E ray travels faster than O ray as the crystal is the negative crystal. This results in a phase difference δ between the two rays emerging from the crystal.



Let the vibrations of the incident plane polarized light make an angle of θ with the optic axis as shown. If A is the amplitude of incident vibrations, the amplitude of extra-ordinary vibrations along PE is is given by $a = A\,cos\theta$ ....(1)
and the amplitude of the ordinary vibrations along PO is given by
$b = A\,sin\theta$ ...(2)
The two rays emerging from the crystal can be represented by two simple harmonic vibrations of different amplitudes having a phase difference δ between them. If f is the frequency of vibrations, then the displacement of E ray vibrations along PE is

$x = a\,sin(\omega t + \delta)$ ..... (3)

The displacement O ray vibrations along PO is given by $y = b\,sin\omega t$ ....(4) where $\omega = 2\pi f$

Equation (4) can be rewritten as $sin\omega t = \frac{y}{b}$ ....(5)

Squaring the above equation $\frac{y^2}{b^2} = sin^2\omega t$ or $\frac{y^2}{b^2} = 1 - cos^2\omega t$

or $cos^2\omega t = 1 - \frac{y^2}{b^2}$ or $cos\omega t = \sqrt{1 - \frac{y^2}{b^2}}$ ......(6)

from equation (3) we have $\frac{x}{a} = sin\omega t\, cos\,\delta + cos\omega t \sin\delta$ ....(7)

substituting the terms of (5) and (6) in (7), we get $\frac{x}{a} = \frac{y}{b}\, cos\,\delta + \sqrt{1 - \frac{y^2}{b^2}}\, \sin\delta$

or $\frac{x}{a} - \frac{y}{b}\, cos\,\delta = \sqrt{1 - \frac{y^2}{b^2}}\, \sin\delta$

squaring the above equation, $\left(\frac{x}{a} - \frac{y}{b}\, cos\,\delta\right)^2 = \left(1 - \frac{y^2}{b^2}\right) sin^2\delta$

simplifying, $\frac{x^2}{a^2} + \frac{y^2}{b^2} cos^2\delta - \frac{2xy}{ab}\, cos\,\delta = sin^2\delta - \frac{y^2}{b^2}\, sin^2\delta$

or $\boldsymbol{\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\, cos\,\delta = sin^2\delta}$ . .....(8)

This is the general equation of an ellipse. The light emerging from the crystal is in general elliptically polarized.

Special cases:

Case (i) When the phase difference is $\delta = 0, 2\pi, 4\pi, \ldots\ldots$, $\cos\delta = 1$ and $\sin\delta = 0$.

Thus equation (8) becomes $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$ or $\left(\boldsymbol{\frac{x}{a} - \frac{y}{b}}\right)^2 = 0$

or $\boldsymbol{\frac{x}{a} = \frac{y}{b}}$ ***or*** $\boldsymbol{y = \left(\frac{b}{a}\right) x}$ This equation represents a straight line with a positive slope and thus the emergent ray is plane polarized along the same plane as that of original vibrations of light.

When the phase difference is $\delta = \pi, 3\pi, \ldots\ldots$, $\cos\delta = -1$ and $\sin\delta = 0$.

Thus equation (8) becomes $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2xy}{ab} = 0$ or $\left(\boldsymbol{\frac{x}{a} + \frac{y}{b}}\right)^2 = 0$

or $\boldsymbol{\frac{x}{a} = -\frac{y}{b}}$ ***or*** $\boldsymbol{y = -\left(\frac{b}{a}\right) x}$

This equation represents a straight line with a negative slope and thus the emergent ray is plane polarized along the same plane as that of original vibrations of light.

Case (ii) : When the phase difference is $\delta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ldots$ $\cos\delta = 0$ and $\sin\delta = 1$

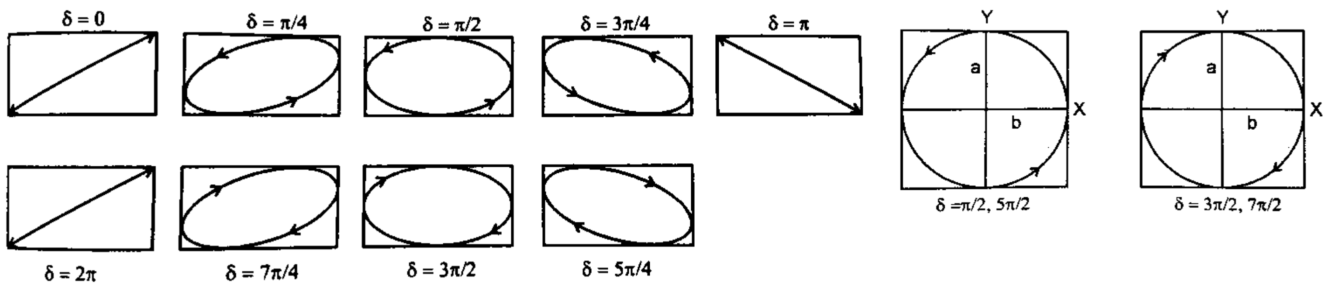
Thus equation (8) becomes $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \mathbf{1}$ This equation represents equation of an ellipse. Thus the emergent light is elliptically polarized.

Case (iii) : When the phase difference is $\delta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ldots$ $\cos\delta = 0$ and $\sin\delta = 1$

Also, if $\theta = 45^0$, then $a = A\,cos45^0$ and $b = A\,sin45^0$. Thus $\boldsymbol{a = b}$

Thus the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \mathbf{1}$ is $\frac{x^2}{a^2} + \frac{y^2}{a^2} = \mathbf{1}$ or $\boldsymbol{x^2 + y^2 = a^2}$

This equation represents a circle of radius $\boldsymbol{a}$**.** The emergent light vibrations from the crystal are circularly polarized. All the special cases are represented by the diagrams as shown below.



Quarter wave plate

A plate of doubly refracting uniaxial crystal (quartz or calcite) whose refracting faces are cut parallel to the direction of optic axis and whose thickness is such as to produce a phase difference of $\frac{\pi}{2}$ or a path difference of $\frac{\lambda}{4}$ between the ordinary and extraordinary rays is called quarter wave plate.

Let d be the thickness of the quarter wave plate and n_o and n_e be the refractive indices of the medium for the ordinary and extraordinary rays respectively. For normal incidence, the path difference between the extraordinary and the ordinary rays is given by

$path\ difference = d\ (n_o - n_e)$ for the negative crystals and

$path\ difference = d\ (n_e - n_o)$ for the positive crystals.

But the path difference for a quarter wave plate is equal to $\frac{\lambda}{4}$.

Thus for negative crystals $d\ (n_o - n_e) = \frac{\lambda}{4}$ or $\boldsymbol{d = \frac{\lambda}{4(n_o - n_e)}}$

Similarly for the positive crystals $d\ (n_e - n_o) = \frac{\lambda}{4}$ or $\boldsymbol{d = \frac{\lambda}{4(n_e - n_o)}}$

A quarter wave plate introduces a phase difference of $\Delta\phi$ between E -O- ray and O- ray given by $\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2} = 90^0$.

If the thickness of the plate is such that $d\,(n_o - n_e) = (2m+1)\frac{\lambda}{4}$ where $m = 0, 1, 2, \ldots.$ Then the plate still acts as a quarter wave plate for all these integers.

A quarter wave plate is used to produce and detect circularly and elliptically polarized light. If the plane polarized light incident on the quarter wave plate with its vibrations making an angle of 45^o with the optic axis, then the emergent light is circularly polarized. For angles other than 45^o, it is elliptically polarized.

Half wave plate

A plate of doubly refracting uniaxial crystal (quartz or calcite) whose refracting faces are cut parallel to the direction of optic axis and whose thickness is such as to produce a phase difference of π or a path difference of $\frac{\lambda}{2}$ between the ordinary and extraordinary rays is called quarter wave plate.

Let d be the thickness of the half wave plate and n_o and n_e be the refractive indices of the medium for the ordinary and extraordinary rays respectively. For normal incidence, the path difference between the extraordinary and the ordinary rays is given by

$path\ difference = d\,(n_o - n_e)$ for the negative crystals and

$path\ difference = d\,(n_e - n_o)$ for the positive crystals.

But the path difference for a half wave plate is equal to $\frac{\lambda}{2}$.

Thus for negative crystals $d\,(n_o - n_e) = \frac{\lambda}{2}$ or $\boldsymbol{d = \frac{\lambda}{2(n_o - n_e)}}$

Similarly for the positive crystals $d\,(n_e - n_o) = \frac{\lambda}{2}$ or $\boldsymbol{d = \frac{\lambda}{2(n_e - n_o)}}$

A half wave plate introduces a phase difference of $\Delta\phi$ between E -O- ray and O- ray given by $\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi = 180^0$.

If the thickness of the plate is such that $d\,(n_o - n_e) = (2m+1)\frac{\lambda}{2}$ where $m = 0, 1, 2, \ldots.$ Then the plate still acts as a half wave plate for all these integers.

When a incident plane polarized light is incident on a half wave plate, the emergent light is also plane polarized for all orientations of the plate with respect to the plane of vibration of incident light.

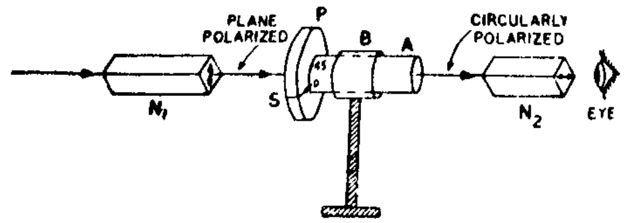
Quarter wave plate and the half wave plate are called retarding plates as they retard one of the beams.

Production of plane polarized, circularly and elliptically polarized light

Plane polarized light

Production : A unpolarised light is allowed pass through a Nicol prism. The light is split in to ordinary and extraordinary rays. The ordinary ray is totally reflected by the Canada balssm material in the Nicol prism. Only extraordinary ray with vibrations parallel to principal section of Nicol prism emerges which is plane polarized light.

Detection : Plane Polarised Light beam is allowed to fall on Nicol prism. The Nicol prism is rotated. The intensity of emitted light gradually decrease and become zero at two positions in each rotation. That is, the intensity varies from zero to maximum. Then the incident light is said to be plane polarized.



Circularly polarized light

Production : To produce circularly polarized light, the two waves vibrating at right angles to each other having the same amplitude and time period should have a phase difference of $\frac{\pi}{2}$ or a path difference of $\lambda/4$.

1. The experimental arrangement is as shown. A beam of monochromatic light falls on the Nicol prism N_1 . The emergent light is plane polarized.
2. When another Nicol prism N_2 is placed at a suitable distance in the path of polarized light, It is rotated till the field of view is dark. Now the two nicols are crossed.
3. Now a quarter wave plate is introduced between the prisms. The field of view is not dark. The quarter wave plate is rotated so that the field of view becomes dark. At this position it is observed that polarized light falling on the plate has its vibrations parallel to optic axis of the plate and perpendicular to N_2.
4. Now the quarter wave plate is rotated through 45^0 so that the vibrations of light falling on the plate make an angle of 45^0 with the optic axis. Now the amplitudes of vibrations of the two rays are equal and there is a phase difference of $\frac{\pi}{2}$ between them. This results in circularly polarized light.

Detection :

1. The light beam is allowed to fall on a Nicol prism. If on rotation of Nicol prism the intensity of emitted light remains same, then light is either circularly polarised or unpolarised.
2. To differentiate between unpolarised and circularly polarised light, the light is first passed through quarter wave plate and then through Nicol prism.

3. If beam is circularly polarised then after passing through quarter wave-plate an extra difference of $\lambda/4$ is introduced between ordinary and extraordinary component and gets converted into plane polarised. Thus on rotating the Nicol, the light can.be extinguished at two plates.
4. If, on the other hand, the beam is unpolarised, it remains unpolarised after passing through quarter wave plate and on rotating the Nicol, there is no change in intensity of emitted light.

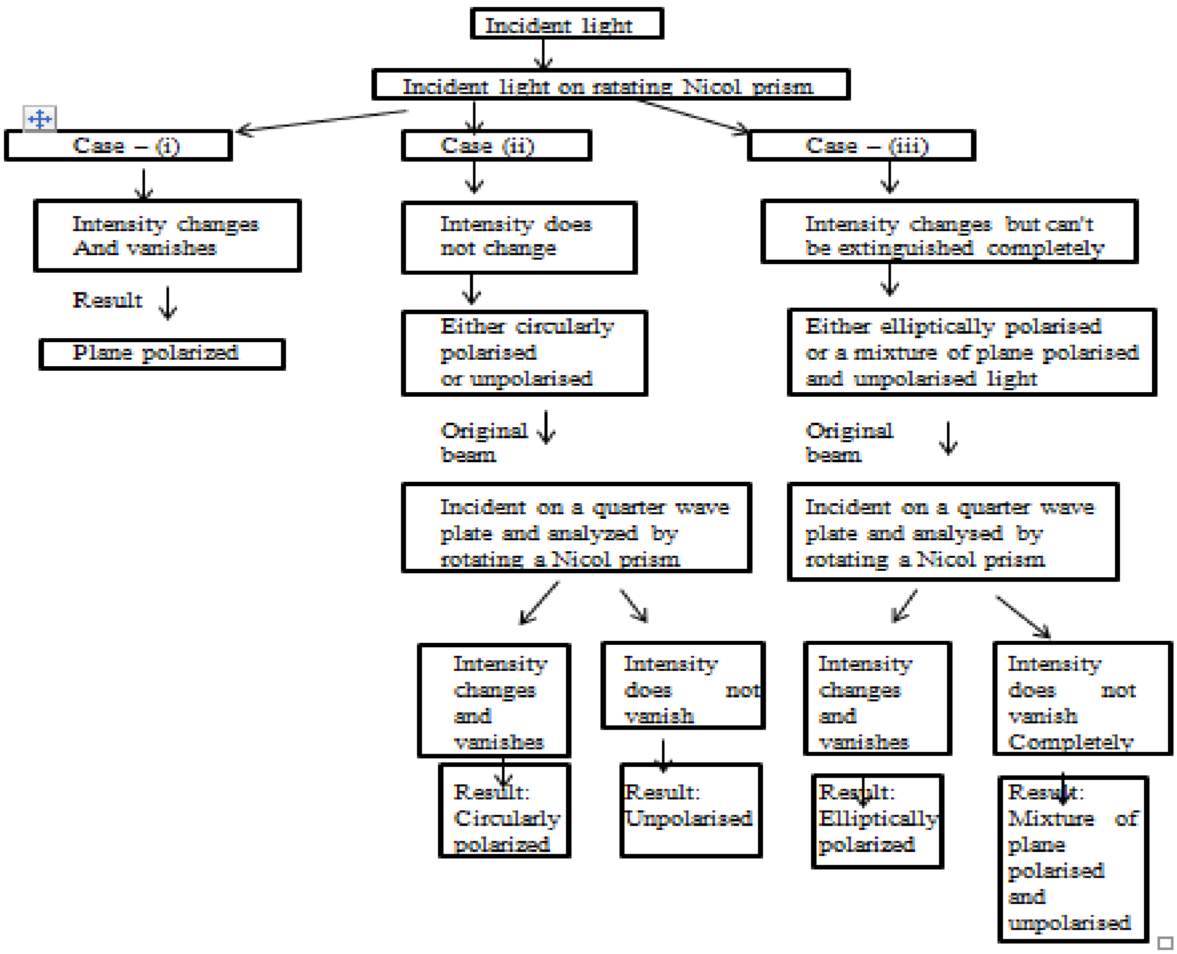
Elliptically polarized light: **Production :**

1. To produce elliptically polarized light, the two waves vibrating at right angles to each other having the unequal amplitude should have a phase difference of $\frac{\pi}{2}$ or a path difference of $\lambda/4$.
2. The experimental arrangement is same as above. A beam of monochromatic light falls on the Nicol prism N_1 . The emergent light is plane polarized.
3. When another Nicol prism N_2 is placed at a suitable distance in the path of polarized light, It is rotated till the field of view is dark. Now the two nicols are said to be crossed.
4. Now a quarter wave plate is introduced between the prisms. The field of view is not dark. The quarter wave plate is rotated so that the field of view becomes dark. At this position it is observed that polarized light falling on the plate has its vibrations parallel to optic axis of the plate and perpendicular to N_2.
5. Now the quarter wave plate is rotated through an angle other than 45^0 so that the vibrations of light falling on the plate make an angle other than 45^0 with the optic axis. Now the amplitudes of vibrations of the two rays are unequal and there is a phase difference of $\frac{\pi}{2}$ between them. This results in elliptically polarized light.

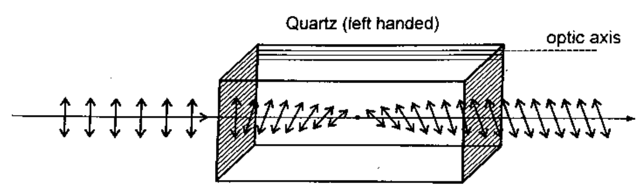
Detection :

1. The light beam is allowed to fall on Nicol prism. If on rotation of Nicol prism, the intensity of emitted light varies from maximum to minimum, then light is either elliptically polarised or a mixture of plane polarized and unpolarised.
2. To differentiate between the two, the light is first passed through quarter wave plate and then through Nicol prism.
3. If beam is elliptically polarised, then after passing through quarter wave plate, an extra path difference of $\lambda/4$ is introduced between O-ray and E-ray and get converted into plane polarized
4. Thus, on rotating the Nicol, the light can be extinguished. If, on the other hand, beam is mixture of polarised and unpolarised it remains mixture after passing through quarter wave plate and on rotating the Nicol intensity of emitted light varies from maximum to minimum.

Detection of types of polarized light in general



Optical Activity



The property of a substance by virtue of which it rotates the plane of polarization the light incident on it is called **optical activity**. The substances having this property are called optically active substances. Eg. Quartz, sugar solution, sodium chlorate, quinine ete…

There are two types of optically active substances.

1. **Dextro-rotatory** or right handed substances are those which rotate the plane of polarisation in the clockwise direction as seen from the emergent side. Eg. Cane sugar.
2. **Laevo-rotatory o**r left handed substances are those which rotate the plane of polarisation in the anticlockwise direction as seen from the emergent side. Eg. Fruit sugar.

Specific rotation or Specific Rotatory Power

The angle of rotation produced by an optically active substance depends on

(1) thickness of a solid substance or length of a solution,

(2) density of a solid substance or concentration of a solution,

(3) temperature

(4) wavelength of light used.

For an optically active solid at a given temperature and for a given wavelength the angle of rotation produced (θ) is directly proportional to the thickness (t)

That is $\theta \propto t$ or $\theta = S t$

The constant S is called the specific rotation or specific rotatory power of the substance. It is given by $S = \frac{\theta}{t}$. Its unit is radm^{-1}.

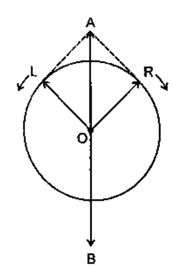
Specific rotation of an optically active solid substance is defined as the angle of rotation of the plane of polarisation produced by the substance of unit thickness a given temperature and for a given wavelength of light.

In case of an optically active solution, the angle of rotation produced (θ) is directly proportional to (1) the length of the solution (L) and (2) concentration of the solution (C), provided the temperature and wavelength of light remain the same.

That is $\theta \propto LC$ or $\theta = SLC$ .The constant S is called the specific rotation or specific rotatory power of the solution and is given by $S = \frac{\theta}{LC}$. Its unit is rad m^2 kg^{-1}.

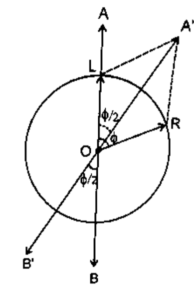
The specific rotatory power of an optically active solution is defined as the angle of rotation of the plane of polarisation produced by the solution of unit length and unit concentration at a given temperature and for a given wavelength of light

Fresnel's theory of optical activity



1. Fresnel theory of optical activity is based on the fact that when plane polarized light is allowed to pass through a crystal along the optic axis, it is split into two circularly polarized vibrations rotating in opposite directions with the same frequency and each with an amplitude half that of the incident light.
2. The velocities of the component wavers are the same in an optically inactive crystal whereas the velocities are different in an optically active crystal.
3. Calcite crystal is not optically active.

4. The two component circularly polarized vibrations move forward with same velocity and therefore have a phase of /2 .
5. Let OL be the component of circularly polarized vector rotating to the left (anti-clockwise) and OR the component rotaing to the right (clockwise). The resultant will be along OA. Thus there is no rotation.
6. Quartz crystals are optically active. The two component circularly polarized vibrations move forward with slightly different velocities and therefore have definite phase difference between them.



7. In case of a right handed quartz crystal, the velocity of clockwise rotation OR is more than the anti-clockwise rotation OL. Thus in a dextro-rotatory quartz crystal, the resultant of OR and OL will be along OA'. The incident plane polarized vibrations are along AB. Thus the plane of vibration of light is rotated through an angle $\phi/2$ on passing through the crystal as shown. This angle of rotation depends on the thickness of the crystal.

Analytical treatment :

In a dextro-rotatory optically active crystal like quartz, the clockwise vibrations travel faster than the anticlockwise vibrations. The clockwise vibrations start from R and anticlockwise starts from L with a phase difference of ϕ as shown in diagram above. The clockwise vibration is given by $x_1 = a\,cos(\omega t - \phi\ )$ and $y_1 = a\,sin(\omega t - \phi\ )$

(ϕ is taken as negative since it is measured in clockwise direction)

The anticlockwise vibration is given by $x_2 = -a\,cos\omega t$ and $y_2 = a\,sin\omega t$

where a is the amplitude of the component waves.

The resultant vibration along the X-axis is $X = x_1 + x_2 = a\,cos(\omega t - \phi\ ) - a\,cos\omega t$

or $X = -2a\,sin\left(\frac{\phi}{2}\right) sin\left(\omega t - \frac{\phi}{2}\right)$ .....(1)

The resultant vibration along the Y direction is $Y = y_1 + y_2 = a\,sin(\omega t - \phi\ ) + a\,sin\omega t$

or $Y = 2a\,cos\left(\frac{\phi}{2}\right) sin\left(\omega t - \frac{\phi}{2}\right)$ .....(2)

The resultant vibrations X and Y are perpendicular to each other, in the same phase. Thus the resultant vibration due to X and Y is plane polarized. Dividing (1) by (2) $\frac{X}{Y} = tan\left(\frac{\phi}{2}\right)$.

The resultant emergent plane polarized light is inclined at an angle $\left(\frac{\phi}{2}\right)$ to the original direction. Thus the emergent plane polarized light is rotated through an angle $\left(\frac{\phi}{2}\right)$ on passing through the crystal and depends on the thickness of the crystal.

If the refractive index of clockwise vibration is n_R and that if anticlockwise vibration is n_L, the optical path difference in passing through a thickness d of the crystal = $(n_L - n_R)d$.

If the wavelength of light is λ, then the

$Phase\ difference\ \phi = \frac{2\pi}{\lambda} \times path\ difference.$ Thus $\phi = \frac{2\pi}{\lambda}(n_L - n_R)d$

$\frac{\phi}{2} = \frac{\pi}{\lambda}(n_L - n_R)d$ or $\frac{\phi}{2} = \frac{\pi}{\lambda}\left(\frac{c}{v_L} - \frac{c}{v_R}\right)d$ or $\frac{\phi}{2} = \frac{\pi c}{\lambda}\left(\frac{1}{v_L} - \frac{1}{v_R}\right)d$

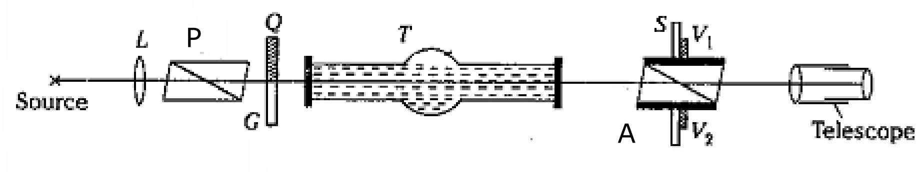
As $= f\lambda = \frac{\lambda}{T}$, we have $\boldsymbol{\frac{\phi}{2} = \frac{\pi d}{T}\left(\frac{1}{v_L} - \frac{1}{v_R}\right)}$

For clockwise rotation (dextrorotatory substance) $v_L > v_R$ and for anticlockwise rotation (laeorotatory substance) $v_R > v_L$. For optically inactive substance $v_L = v_R$. Thus $\frac{\phi}{2} = 0$.

Laurent's half shade polarimeter:

A polarimeter is an instrument used to measure the specific rotatory power of an optically active solution.

Construction

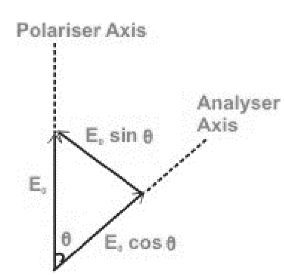


It consists of a glass tube T placed between two Nicol prisms P called polarizer and A the analyzer. A half shade device GQ is placed between polarizer and the tube. It has two semicircular plates one made of quartz and the other made of glass.

A telescope is used to observe the light emerging from the analyser. *S* is a circular scale fixed to the analyser using which angle of rotation can be determined.

Working :

1. The glass tube is filled with distilled water (without air bubbles). Light from a source is made to fall on the polarizer. The beam emerging from P is plane polarised. This light falls on the half shade arrangement.
2. The plane of vibration of light passing through quartz is rotated while the plane of vibration of light passing through glass remains unaltered. Hence the field of view seen through the telescope will have two distinct halves.
3. The principal section of analyzer A is made symmetric with respect_to the planes of vibration of light passing through quartz and glass so that both the halves appear equally bright. The reading R_0 in the analyzer is noted.
4. Now the tube is filled with optically active solution of known concentration (C). The solution rotates the planes of vibration of both the halves through the same angle. Thus the brightness of the two halves will be different again. Analyser is rotated until the two halves have the same brightness and the reading *R* is noted. $R \sim R_o = \theta$ gives the angle of rotation. The length of the tube L is measured. Specific rotatory power is calculated using $S = \frac{\theta}{LC}$.



Note : Malus Law - According to malus, when completely plane polarized light is incident on the analyzer, the intensity I of the light transmitted by the analyzer is directly proportional to the square of the cosine of angle between the transmission axes of the analyzer and the polarizer. i.e. $\mathbf{I \propto \cos^2\theta}$

Descriptive Questions

1. (a) What is meant by polarization? Explain.
   (b) Distinguish between plane of polarization and plane of vibration.
2. (a) Explain polarization by reflection. What is Brewster's law.
   (b)Prove that the reflected and refracted rays are at right angles to each other whenever the reflected light is completely plane polarized.
3. (a) Explain the phenomenon of polarization by double refraction.
   (b) Describe Huygen's explanation of double refraction in uniaxial crystals.
4. (a) Describe the Huygen's explanation of double refraction for oblique incident beam normal to the surface of a doubly refracting crystal with optic axis inclned to the surface of the crystal.
   (b) Describe the Huygen's explanation of double refraction for a parallel incident beam normal to the surface of a doubly refracting crystal with optic axis inclined to the surface of the crystal.
5. Give the theory of retarding plates and discuss the special cases.
6. (a) Explain the construction, principle and use of a quarter wave plate.
   (b) Derive equation for minimum thickness of a quarter wave plate for light of wavelength λ.
7. (a) Explain the construction, principle and use of a half wave plate.
   (b) Derive an expression for the thickness of a half wave plate. Distinguish between quarter wave plate and half wave plate.
8. (a) How can circularly polarized light be produced and detected? Explain.
   (b) How can elliptically polarized light be produced and detected? Explain.
9. (a) What is optical activity?
   (b) Describe Fresnel's explanation for the optical activity. Derive an expression for the optical activity. Derive an expression for the angle of rotation produced by an optically active substance.
10. (a) Distinguish between dextro rotatory and laevo rotatory optically active substances.
    (b) Describe Laurent's half-shade polarimeter method of determining the specific rotation of sugar solution.

Numerical Problems

1. Plane polarized light is incident on a piece of quartz-cut parallel to the axis. Find the least thickness for which the ordinary and extra-ordinary rays combine to form plane polarized light. Given n_0 = 1.5442, n_e = 1.5533 and λ = 5000 Å.

[$d\,(n_e - n_o) = \frac{\lambda}{2}$ or $d = \frac{\lambda}{2(n_e - n_o)} = 27.5\,\mu m$]

2. A 0.25mm thick crystal plate cut parallel to optic axis serves as quarter wave plate for wavelength $\lambda = 530$ nm,. At what other wavelengths of visible spectrum will it also serve as a quarter wave plate? Assume that difference in refractive indices (n_e - n_o) = 0.009 is constant for all visible spectrum. [$d\,(n_o - n_e) = (2m+1)\frac{\lambda}{4}, thus\ \ d\,(n_o - n_e) = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$. The other possible values of λ are $\frac{\lambda_1}{4} = \frac{3\lambda_2}{4} = \frac{5\lambda_3}{4}$.. $\lambda_2 = 176.6\,nm,\ \lambda_3 = 106\,nm$]
3. Calculate the thickness of a mica sheet required for making a quarter wave plate. Given refractive indices for ordinary and extra – ordinary rays in mica are 1.586 and 1.592, $\lambda =$ 546 nm. [$d = \frac{\lambda}{4(n_e - n_o)} = 22.75$ μm]
4. A plane polarized light is incident perpendicularly in a quartz plate cut with faces parallel to optic axis. Find the thickness of quartz plate, which introduces phase difference of 60^0 between O- and E- rays. Given $\lambda = 540$ nm. Assume that difference in refractive indices (n_e - n_o) = 0.009 is constant for all visible spectrum

   [$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$ or $\Delta x = \frac{\lambda}{2\pi} \times \Delta\phi = \frac{\lambda}{2\pi} \times \frac{\pi}{3} = \frac{\lambda}{6}$. Thus $d = \frac{\lambda}{6(n_e - n_o)} = 10\mu m$]

5. Plane polarized light passes through a quartz plate with its optic axis parallel to the faces. Calculate the least thickness of the plate for which the emergent beam (i) will be plane polarized and (ii) will be circularly polarized.
   Given $n_o = 1.544, n_e = 1.553\ and\ \lambda = 5000$Å.
   [For plane polarized light $d = \frac{\lambda}{2(n_e - n_o)} = 27.77$ μm For circularly polarized light $d = \frac{\lambda}{4(n_e - n_o)} = 13.88$ μm]
6 Calculate the least thickness of a calcite plate which convert plane polarized light into circularly polarized light, if the refractive indices for sodium light are respectively 1.658 and 1.486 for the ordinary and extraordinary ray. Given $\lambda = 5893$Å. [$d = \frac{\lambda}{4(n_e - n_o)} = 0.856$ μm]
7. A half wave plate is constructed for a wavelength of 6000Å. For what wavelength does it work as a quarter wave plate
   [$\frac{\lambda_1}{4(n_e - n_o)} = \frac{\lambda_2}{4(n_e - n_o)}$ or $\lambda_2 = 2\,\lambda_1 = 12000$ Å
8 Calculate the rotation of plane polarization in a substance of unit thickness of wavelength 5890Å. The difference between the refractive indices for right and left circularly polarized lights in the substance is 7.62×10^{-8}. [$\boldsymbol{Ans: 0.798\,rad}$]

   [$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$ The path difference $\Delta x = d(n_e - n_o)$
   Thus phase difference $\Delta\phi = \frac{2\pi}{\lambda} \times d(n_e - n_o) = 0.8124\,rad$ or $0.8124 \times \frac{180}{\pi} = 46.57^0$]

9 The rotation of the plane of polarization is 40° in a substance of thickness 2 m. If the difference between the refractive indices for left and right circularly polarized light in the substance is 3.2738×10^{-8}, calculate the wavelength of the light used.

[$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x \quad or \quad \Delta x = \frac{\lambda}{2\pi} \times \Delta\phi \quad As\ \Delta x = d(n_e - n_o), \quad d(n_e - n_o) = \frac{\lambda}{2\pi} \times \Delta\phi$

$\lambda = d(n_e - n_o)\frac{2\pi}{\Delta\phi} = 589.2\ nm$ (convert deg to rad $40° = 40 \times \frac{\pi}{180}\ rad$)]

10 Sugar solution of concentration 100kg m^{-3} is kept in a polarimeter tube of length 0.22 m. If the specific rotation of sugar is $0.75\,°\,\text{kg}^{-1}\text{m}^2$, calculate the rotation of the plane of polarization.

[$S = \frac{\theta}{L\,C} \quad or \quad \theta = S\,L\,C = 100 \times 0.22 \times 0.75 \times \frac{\pi}{180} = 0.287\,rad = 16.49^0$]

11 The rotation of the plane of polarization in a certain substance is $40°\text{m}^{-1}$ for light of wavelength 5890 Å. Calculate the difference between the refractive indices for right and left circularly polarized light in the substance.

[Given $\frac{\Delta\phi}{d} = 40^0 m^{-1} = 40 \times \frac{\pi}{180}\ rad\ m^{-1} \qquad \Delta\phi = \frac{2\pi}{\lambda} \times d(n_e - n_o) \ or \ (n_e - n_o) = \frac{\lambda}{2\pi} \times \frac{\Delta\phi}{d}$

$(n_e - n_o) = 6.544 \times 10^{-8}$]

12 A 0.02 m long polarimeter tube containing a certain solution of concentration 200kg m^{-3} produces an optical rotation of 24°. Find the specific rotation of the solution

[$S = \frac{\theta}{L\,C} = 0.01046\,rad kg^{-1} m^2$ ($\theta = 24 \times \frac{\pi}{180}\ rad$)]

13 Determine the concentration of solution of length 0.25 m which produces an optical rotation of 30°. The specific rotation of the solution is $0.0209\ \text{rad m}^2\ \text{kg}^{-1}$

[$C = \frac{\theta}{L\,S} = 100.2\ kg\ m^{-3}$ ($\theta = 30 \times \frac{\pi}{180}\ rad$)]

14 A polarimeter tube of length 0.2 m contains 20% sugar solution. The specific rotation of the solution is 0.6 degree per metre per unit concentration. Calculate the optical rotation produced by the solution.

[$\theta = S\,L\,C = 0.6 \times \frac{\pi}{180} \times 0.2 \times C$ where $C = \frac{20}{100}\frac{g}{cc} = \frac{20}{100}\frac{10^{-3}kg}{10^{-6}m} = 200kgm^{-3} \quad \theta = 24^0$]

15 Calculate the length of the solution of concentration 10% which produces an optical rotation of 35^0. The specific rotation of the solution is $0.0305\ \text{rad m}^2\ \text{kg}^{-1}$.

[$L = \frac{\theta}{S\,C} = 35 \times \frac{\pi}{180} \times \frac{1}{0.0305 \times C} = 0.2\,m$ ($C = \frac{10}{100}\frac{g}{cc} = \frac{20}{100}\frac{10^{-3}kg}{10^{-6}m} = 100kgm^{-3}$)]