STUDIES ON HEAT AND MASS TRANSFER PROBLEMS IN SATURATED POROUS MEDIA

Report of the work done under the UGC Minor Research Project in Physics

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DECLARATION

I herby declare that the matter embodied in this report is the result of the investigations carried out by me independently in the Department of Physics, Vijaya College, R V Road, Basavanagudi, Bangalore 560 004. No part of the subject matter presented in this report has been submitted for the award of any Degree, Diploma, associateship, fellowship, etc of any University or Institute

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NagarajuPuttabasavasetty

Content

CHAPTER

CHAPTER 1

INTRODUCTION

1.1 Need for the study

The severity of the energy and ecology problems facing man has incited him to look for alternate source and could be one of the reasons for growing interest in buoyancy driven transport processes. Heat transfer is commonly encountered in engineering systems and other aspects of life and one doesn't need to go very far to see some application areas of heat transfer. The human body itself is constantly rejecting heat to its surroundings and human comfort is closely tied to the rate of this heat rejection. As a matter of fact one will control his/her heat transfer rate by adjusting the clothes to the environmental conditions. Heat transfer plays a major role in the design of many other devices such as car radiators, solar collectors, various components of power plant and even space craft. Natural convection in a porous medium is important in many technological applications and is increasing in importance with the growth of interest in geothermal energy. One of the motivations for study of convection in a porous medium comes from geophysics. Thermal convection problems in porous media occur in a broad spectrum of disciplines ranging from chemical engineering to geophysics. Applications include heat insulation by fibrous materials, spreading of pollutants and convection in the earth's mantle. Porous media analysis is used to model the heat and mass transfer which occurs during the storage of agricultural products such as fruits, vegetables and grains. Transport of solutes in porous media is of interest in ground water hydrology, disposal of wastes, petroleum reservoir engineering, mineral extraction and oil recovery system, recovery of water for drinking and irrigation, salt water encroachment into fresh water reservoirs, in biophysics-life processes such as flow in the lungs and kidneys. The spreading of contaminants in environmental flows is a major conceptual and practical challenge [1] these days. Transport of solutes takes place not only just below the surface of the earth, but also deep in the earth. As suggested by Mc Nbb [2], the brine (saline fluid) released from the 'hot plate' beneath the hydrothermal system may also get heated up giving rise to fluid saturated with solutes. The earth's crust being at very high temperature and great pressure melts and becomes part of the mantle. The modern trend towards miniaturization of devices requires a better understanding of heat and mass transfer phenomena in small dimensions. Several applications related to porous media require a detailed analysis of convective heat transfer in different geometrical shapes, orientations and configurations. The versatility of this field is such that it covers large number of practical aspects of human life such as petrochemical refineries, thermal insulation of buildings, solidification of alloys, drying process, catalytic and chromatographic reactions, packed absorption and distillation towers, ion exchange column, packed filters and pebble-type heat exchangers. The design of these systems is governed by the pressure drop, fluid flow, and heat and mass transfer processes in the packed bed arrangement. Even though a number of articles are published on this topic, still many aspects remain unexposed. Thus the present study considers natural convection in a porous medium and determines the effects of different modes of heating from below the rectangular enclosure packed with spherical balls by keeping the upper boundary closed as well as open.

1.2 Objective of the study

The objectives of the present work are the following:

i) To understand the mechanism of free or Natural convection in a rectangular cavity filled with porous media (different kind of solid particles)

ii) To investigate the effect of porous media on Natural convection as compared to the absence of porous media

iii) To study the importance of variation of transport properties like thermal conductivity and permeability in a rectangular cavity packed with porous media

iv) To study the effect of vertical orientation in comparison with horizontal orientation of the rectangular cavity in the presence as well as in the absence of porous medium

1.3 Review of the literature

The phenomenon of Natural convective heat transfer in a confined rectangular cavity has been studied both experimentally and theoretically (analytical and/or numerical) by a number of researchers [3-8]. In a series of investigations, Blythe et al [9,10] discussed the large Rayleigh number limit for a fixed aspect ratio and presented some new scaling laws for the problem of natural convection in a rectangular cavity. An experimental investigation of the problem by Tong et al [11] led to the authors to conclude that the heat transfer could be minimized by filling the enclosure partially with a porous medium rather than filling it completely. A more practical case where there is no impermeable partition between the fluid and the porous medium was considered experimentally by Beckermann et al [12]. The literature on Natural convection in porous medium heated from below includes early work by Lapwood [13], Elder [14] and Prasad et al [15] to determine the conditions for the onset of convection. Natural convection on very large scale occurs both in the atmosphere and within the earth crust. The flow phenomenon has long been of interest to geophysicists, meteorologists and petroleum reservoir engineers. The engineering applications of open cavities are important [16-17] for the case of fire spread in rooms, solar thermal central receiver systems, connections between reservoirs and nuclear waste repositories. One of the reasons for these investigations can be traced to the difficulty in specifying the boundary conditions (due to its significantly more complex geometry). It should be mentioned that there has been many investigations on the **two dimensional** natural convection in porous enclosures [18-19], but they are not well received because of their complexity. An experimental investigation of natural convection in liquid saturated confined porous medium in inclined plane was studied by Kaneko et al [20] and Bories et al [21]. In recent years, engineers have focused their attention on thermal insulation and heat storage using porous media [22-25]. Generally, the onset of natural convection in porous medium is somewhat complex due to the restrictions imposed by the solids and hence large temperature gradients are normally required to start convective currents. The results of the earlier work (Experimental/Theory) suggested that Darcy's law must be used to describe flow in porous media and Navier -Stokes equations for flow in fluid layers. From the survey of the applications pertaining to geothermal systems, the following points are evident:

1. The heat/mass supplied from underneath generates the buoyancy force and is responsible to transfer such enormous amount of power and energy

2. Heat/mass transfer takes place from great depth to shallow depths

3. Exploitation of energy is possible only in high permeability regions

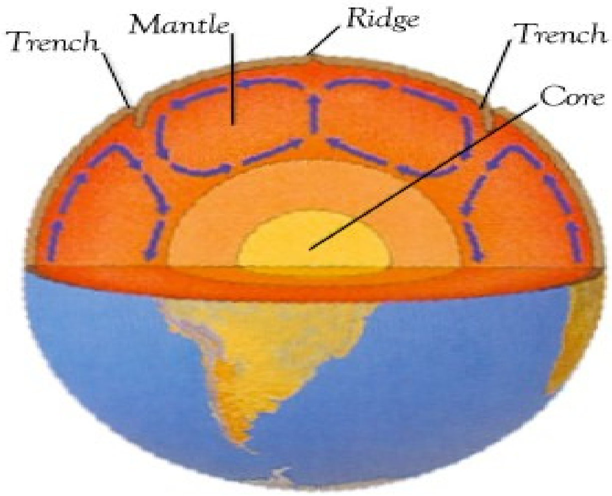

Figure 1 Convection Currents in the earth

In recent years there has been considerable interest given to the study of convection problems in a porous medium because of its importance in many industrial and geophysical problems in cryogenic industry.

1.4 Heat transfer

Heat transfer is the science of spontaneous irreversible process of heat propagation in space and involves the exchange in internal energy between individual elements in the region of the medium considered. There are three basic mechanisms of heat transfer. They are conduction, convection and radiation. The combined process of heat transport by convection and conduction is referred to as convective heat transfer. In convection itself, there are different types. They are convection in horizontal layers and convection in vertical layers (free convection), forced convection and mixed convection. The subject of engineering analysis is usually the transfer of heat by convection between a stream of liquid or gas and a solid surface.

In what follows is the explanation for convection.

Convection: Convection is the transfer of heat energy by movement of heated fluid particles. This movement occurs into a fluid or within a fluid and can't happen solids. In reality this is a combination of diffusion and bulk motion of molecules. In natural convection, a fluid surrounding a heat source receives heat, becomes less dense and raises, the surrounding cooler fluid then moves to replace it. This cooler fluid is then heated and the process continues, forming convection current. The driving force for natural convection is buoyancy, a result of difference in fluid density when gravity or any type of acceleration is present in the system. Thus, the natural convection is caused by buoyancy forces due to density difference caused by temperature variations in the fluid. Boiling or condensing processes are referred to as a convection heat transfer processes. Forced convection occurs when a fluid flow is induced by an external force, such as a pump, fan or a mixer to propel the fluid and create an artificial induced convection current. In some heat transfer systems, both natural and forced convection contribute significantly to the rate of heat transfer.

To calculate the rate of convection between an object and the surrounding fluids, engineers employ the heat transfer co-efficient (h). Unlike the thermal conductivity (λ), the heat transfer co-efficient is not a material property. The heat transfer co-efficient depends on the geometry, fluid, temperature, velocity and other characteristics of the system in which convection occurs. Therefore, the heat transfer co-efficient must be derived or found experimentally for every system analyzed. The heat transfer per unit surface through convection was first described by Newton and the relation is known as the Newton's law of cooling.

The rate of convection heat transfer is observed to be proportional to the temperature difference, and is expressed as

$$\frac{dQ}{dt} = q = hA(T_s - T_\infty) \qquad (1.1)$$

Where 'h' is the convection heat transfer co-efficient in W/m^2K, T_s is the temperature at the hot surface, T_∞ is the temperature of the bulk fluid at infinite distance from the hot surface and A is the surface area through which convection heat transfer takes place.

Benard convection: It is known that the free or natural convection takes place in the still fluid due to its heating by the contact with the hot body. It was Benard (1900) who has studied the free convection extensively by both experimentally and theoretically. The theoretical work has provided most accurate information about when flow occurs, on the other hand most of our knowledge about the detailed flow patterns from experiment. In Benard convection, initially there is equilibrium, which is later upset by various factors, leading to unstable equilibrium. The study of onset of motion in free convection is the subject of stability of flows.

In convection involving vertical layers e.g., flow in a slot involves a fluid layer between vertical plates. In this case the fluid next to the hot wall rises and that next to the cold wall falls, thus the motion always exists. When the flow is established a in steady pattern, then the continuous release of potential energy is balanced by viscous dissipation of mechanical energy. The potential energy is provided by heating at the bottom and cooling at the top. The onset of motion is readily detected by its effect on heat transfer, which is defined as

$$q = hA(T_2 - T_1) \qquad (1.2)$$

These symbols have the same meaning as explained before. Rayleigh (1916) developed the theory for the instability. According to Rayleigh the gravity pulls the cooler denser fluid from the top to the bottom but the buoyancy force pushes the lighter fluid upwards. Thus, the gravitational force between the fluid layer is opposed by the viscous damping force of the fluid. The balance of these two forces is expressed by a non dimensional parameter called the Rayleigh number. Benard convection occurs only when certain criteria is fulfilled. This criteria is expressed in terms of a non-dimensional number known as Rayleigh number

$$R_a = \frac{g\beta(T_2 - T_1)H^3}{\upsilon\alpha} \qquad (1.3)$$

Where β- thermal expansion co-efficient, υ- kinematic viscosity, α- thermal diffusivity ($=\lambda/\rho c_p$)

Convection Cells: A convection cell, also known as a Benard cell is a characteristic fluid flow pattern in many convection systems. A rising body of fluid typically loses heat because it encounters a cold surface and it exchanges heat with colder liquid through direct exchange. In the example of earth's atmosphere, because it radiates heat, the fluid loses heat and hence becomes denser than fluid underneath it, which is still rising. Since it can't descend through the rising fluid, it moves to one side. At some distance, its downward force overcomes the rising force beneath it, and the cycle repeats itself.

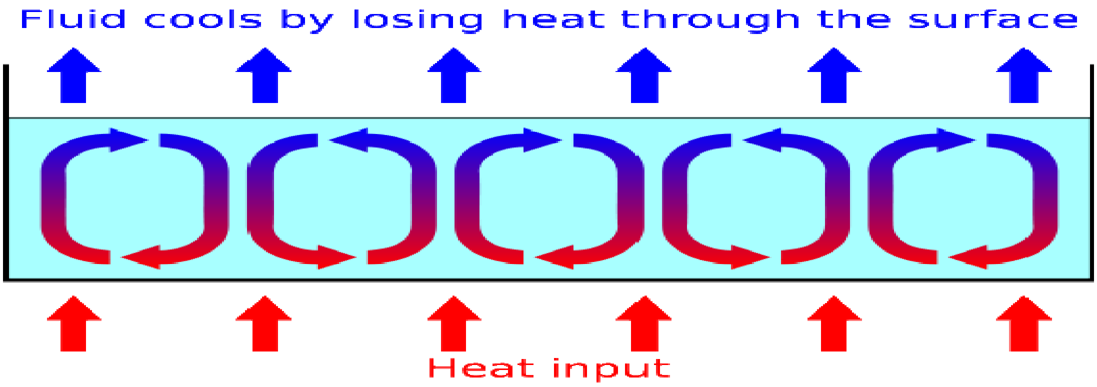

Figure 2 Convection Cells

The subject of engineering analysis is usually the transfer of heat by convection between a stream of liquid or gas and solid surface. The process of heat transfer can be studied either by experiment or by theory. But, the experimental approach is advantageous over theoretical means. As a matter of fact, the experimental approach deals with the actual physical system and it produces what is required within the limits of experimental error. But this approach is expensive, time consuming and often impractical. On the other hand, the theoretical approach (analytical/ numerical) is fast and inexpensive but the results obtained are subject to the accuracy of the assumptions and idealizations made in the analysis.

Porous media: A porous medium is a solid with voids in it. Usually the number of holes or pores is sufficiently large that a volume average is needed to calculate its pertinent properties. Pores which occupy some definite fraction of the bulk volume is called voids. The interconnected pores are very important because they are the ones that affect the flow.

The matrix of a porous medium is the material in which the pores are imbedded. An extremely large array of materials constitute porous medium. Porous media are classified as consolidated and unconsolidated. Examples for consolidated porous media are naturally occurring rocks, sandstones, lime stones, concrete, cement, bricks, cloth, etc. Examples for unconsolidated porous media are beach sand, glass beads, catalyst pellets, soil, Gravel and packing such as charcoal.

There are two levels of description of porous media (i) Microscopic level and (ii) Macroscopic level. At the microscopic level, the description is statistical in nature and in terms of 'pore-size' distribution. At the macroscopic level, the media is described in terms of average or bulk properties and their variation at sizes or scales much larger than pores. The bulk properties that relate to pore description are porosity, permeability, dispersion, tortuosity, capillarity, connectivity, relative permeability, adsorption and wettability. Porous media are widely used in high temperature heat exchangers, turbine blades, jet nozzle etc. Porous media are considered to be useful in diminishing the natural convection. In order to make heat insulation of surface more effective it is necessary to study the free and forced convection flow through a porous medium and to estimate its effect in heat and mass transfer.

Flow through porous media: Flow through porous media concerns itself with the study of the motion and equilibrium of fluids under the action of internal and external forces as well as resistance offered by the solid to the fluid. Flow through porous media is of interest in many branches of engineering such as reservoir, hydraulics, soil mechanics, chemical engineering etc. Reservoir engineer is interested in the recovery of oil or gas, the hydrologist interested in the management of water, the soil scientist is concerned with the subsurface water, the chemical engineer is interested in fixed bed reactors used in refinery or chemical plants and mechanical engineers are concerned with the effective way of cooling a system using porous media. Fluid flow through porous media takes place in a number of technical areas including groundwater flows, flows through embankment dams, paper making composites manufacturing, filtering, drying and sintering of iron ore pellets. The typical applications of flow through porous media are: Acquifer studies, catalyst bed testing, chemical leaching studies, chemical transport simulation, contaminant transport studies, filter design, foam flow studies, gas coolant system design, geologic flow simulation, heat exchange design, landfill design and natural gas exploration studies etc. In several of these areas, the flow can be described by Darcy's law. Wooding and Brinkman have modified the Darcy's law, which is used by many physicists on study of convection flow in porous media.

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CHAPTER 2

BASIC EQUATIONS, BOUNDARY CONDITIONS, NONDIMENSIONAL PARAMETERS AND NOMENCLATURE

The present chapter describes the basic equations and non-dimensional parameters used to study convective heat transfer in porous media.

2.1 Basic equations

The transport process of fluid through a porous medium involves two substance namely, the fluid and the porous matrix. Because of the complexity of the microscopic flow in the pores, one has to consider the gross effect of the phenomena represented by a microscopic view applied to the fluid.

When a fluid passes through a porous medium, it is being opposed by the frictional force offered by the stationary porous material and this force of opposition is given by Darcy's law [Darcy, 1856]. Darcy's law is a generalized relationship for flow in porous media, since its discovery, it has been found valid for any Newtonian fluid. According to this law, the rate at which a fluid flows through a permeable substance per unit area is equal to the permeability, which is a property of a substance through which the fluid is flowing times the pressure drop per unit length of flow divided by the dynamic viscosity of the fluid.

$$\vec{V} = -\frac{k}{\mu}\nabla p \qquad (2.1)$$

Where $\vec{V}$ - mean filter velocity, k- permeability of the medium having the dimension of length square, μ-dynamic viscosity and ∇p - pressure gradient. The Darcy's law is valid for slow, viscous flow and whose Reynold's number is less than 1. This is an empirical equation and is valid for creeping flow through an infinitely extended uniform medium. But, for certain applications such as packed bed catalytic reactors the assumption of Darcian flow will not be valid. Therefore, in such cases non-Darcian effects are quite important. Hence, Darcy's law was modified by Brinkman (1947) and Muskat (1946) by introducing the solid boundary and inertial forces. Thus it takes the form

$$\nabla p = -\frac{\mu}{k}\vec{V} + \mu\nabla^2 V \quad (2.2)$$

Where μ/k –Darcy resistance term and $\mu\nabla^2 V$ – the viscous resistance term. The above equation is referred tor as Brinkman equation. Beavers et al [1] have experimentally demonstrated the existence of a shear within the porous medium near a surface where porous medium is exposed to a freely flowing fluid, thus forming a zone of shear influencing fluid flow. Slattery [2] and Tam [3] have demonstrated that equation (2.2) is the suitable governing differential equation for an incompressible creeping flow of Newtonian fluid within an isotropic and homogeneous porous medium. The above equation has been extensively used by Katto and Masuoka [4], Chandrasekhara et al [5], Chandrasekhara and Nagaraju [6-8] and Nagaraju et al [9].

2.2 Navier-Stokes equation

The motion of the incompressible, Newtonian fluid is governed by the Navier-Stokes equation. This equation arise from applying Newton's second law to fluid motion, together with the assumption that the fluid stress is the sum of diffusing viscous term (proportional to the gradient of velocity), plus a pressure term. These are basic governing equations for viscous, heat conducting fluid.

The equation for incompressible fluid flow is

$$\frac{\partial V}{\partial t} + \vec{V}\,.\nabla V = -\frac{\nabla p}{\rho} + \upsilon\nabla^2 V \quad (2.3)$$

Where υ - kinematic viscosity, $\vec{V}$- velocity of the fluid, p - pressure. ρ - fluid density, $\nabla -$ gradient operator.

This equation is a partial differential equation which describes the conservation of linear momentum for a linearly viscous (Newtonian) incompressible fluid flow in the absence of the porous medium. Using the volume average of a divergence theorem, the local volume averages of the differential balance laws for an incompressible, steady flow through a porous medium can be established as [10]

$$\frac{\rho}{\varepsilon} < \left(\vec{V}.\nabla\right)\vec{V} > = -\nabla p + \frac{\mu}{\varepsilon}\nabla^2 V - \frac{\mu}{k} \quad (2.4)$$

Where ρ- represents the fluid density, ε- the porosity, $\langle p \rangle$- average pressure of the fluid, $\vec{V}$- velocity and μ- dynamic viscosity. This equation signifies the conservation of momentum.

Conservation of mass

$$\nabla.\langle V \rangle = 0 \tag{2.5}$$

Conservation of energy

$$\rho c_p \,(V.\nabla)T = \nabla.\lambda_e \nabla \mathrm{T} + \nabla.q_r \tag{2.6}$$

Here, $\vec{V}$- the velocity vector, T- temperature, ρ- the density of the fluid, c_p – the specific heat capacity, λ_e- the effective thermal conductivity, q_r – radiation heat transfer

2.3 Boundary conditions

The necessary boundary conditions on velocity and temperature fields are given in this section. The boundary surface may be either rigid or free. The nature of the boundary surfaces considered in this dissertation are:

(i) Lower surface is rigid and upper surface is free
(ii) Both lower and upper boundary surfaces are rigid

a) Velocity boundary conditions

The conditions on velocities are obtained from mass balance, no-slip condition and the stress principle of Cauchy, depending upon the nature of the boundary surfaces of the fluid.

1) **Rigid boundaries:** A rigid surface is one on which, the no slip conditions is valid. Hence, not only the vertical component of velocity, but also the horizontal component of velocity must vanish.

$$u = v = w = 0 \tag{2.7}$$

In the case of flow through porous media bounded by rigid boundaries at $y = \pm$ H, the no-slip condition implies that the velocity components vanish identically at the boundaries, i.e.,

$$u = v = w = 0 \quad \text{at} \quad y = \pm H \tag{2.8}$$

2) **Free boundaries**: A free surface is one on which the tangential stress does not act. It is evident that the root cause for tangential stress is the gradient of tangential velocities. Therefore for non-existence of tangential stress one requires the condition that the gradient of tangential velocities should vanish, i.e.,

$$\frac{\partial u}{\partial z} = 0, \qquad \frac{\partial v}{\partial z} = 0 \tag{2.9}$$

$$\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \qquad or \qquad \frac{\partial^2 w}{\partial z^2} = 0 \tag{2.10}$$

b) Thermal boundary conditions

If the bounding walls of the fluid layer have high conductivity and large heat capacity, then its temperature would be spatially uniform for steady state condition. In other words, the boundary temperature would be unperturbed by any flow or temperature perturbation in the fluid. Thus 'T' is constant, this is also referred to as isothermal boundary condition.

2.4 Non dimensional parameters

The principle of dimensionless ratio and dynamical similarity plays an important role in the field of fluid mechanics and heat transfer. The fluid motion is usually a rather complex phenomenon and despite the resources of mathematical analysis one has to rely heavily on observation and measurement of flow properties. There are two general methods of obtaining dimensionless parameters associated with the given physical problem, viz., (i) the inspectional analysis and (ii) the dimensional analysis. The dimensional analysis is the most powerful tool to reduce the volume of experiment or computation needed. It is convenient to have a set of symbols which allow for easy reference to the dimensionless co-efficients appearing in the dimensionless equations of motion. For this purpose it is customary to name a few well-known dimensionless groups with their special significance under given conditions of the flow, their algebraic form may differ from case to case, but the physical meaning is the same. They are the following:

(i) Reynolds number, R_e

Reynolds number characterizes the relation between the forces of inertia and viscosity. Indeed, the Reynolds number is obtained by dividing the term accounting for inertia in the equation of flow by the term allowing for friction in the same equation

$$R_e = UL/\upsilon \tag{2.11}$$

(ii) Prandtl number, P_r

Prandtl number is a characteristic property of the fluid rather than flow. In molecular terms P_r denotes the ratio of the molecular diffusivity (υ) of momentum to the molecular diffusivity of energy ($\alpha=\lambda/\rho c_p$)

$$P_r = \upsilon/\alpha \tag{2.12}$$

The prandtl number plays a significant role, and as a link between the velocity and temperature field.

(iii) Peclet number, P_e

Peclet number is the product of two dimensionless terms:

$$P_e = R_eP_r = UL/\alpha \tag{2.13}$$

It is the ratio of the amount of heat transfer by convection to the amount heat transfer by conduction.

(iv) Nusselt number, Nu

The heat transfer per unit area of surface between a fluid and solid boundary is usually expressed in terms of the heat transfer co – efficient, h, defined by $h = q/\Delta T$. 'h' is not a dimensionless quantity, but a dimensionless quantity results if it is compared with effective thermal conductivity, $q_c = -\lambda\nabla T$. The conventional definition of the Nusselt number is

$$Nu = hL/\lambda \tag{2.14}$$

and is essentially the ratio of the total heat transfer to the conductive heat transfer at the boundary.

(v) **Biot number, Bi**

Biot number is defined the ratio convective heat transfer at the surface of the solid body to the conductive heat transfer within the solid body

$$B_i = hL/\lambda \tag{2.15}$$

This number is significantly used in Forensic Sciences for analyzing the time of death of human being. It determines whether or not the temperature inside a solid body will vary significantly in space, while the body heats or cools over time, from a thermal gradient applied to its surface. In general problems involving small Biot numbers (much smaller than 1) are thermally simple due to uniform temperature fields inside the body. Biot numbers much larger than 1 are more difficult due to non-uniformity of temperature fields within the object. Also it can be applied to HVAC (heating, ventilating and air conditioning or building climate control) to ensure more nearly instantaneous effects of a change in comfort level setting.

(vi) Grashof number, Gr

It is a dimensionless number which is defined as the ratio of the buoyancy to viscous force acting on a fluid. It frequently arises in the study of situations involving natural convection.

$$\text{Gr} = \frac{H^3 g\beta(T_2 - T_1)}{\upsilon^2} \qquad (2.16)$$

This number provides the main criterion in determining whether the fluid flow is laminar or turbulent in natural convection.

(vii) Rayleigh number, Ra

It is a dimensionless number associated with buoyancy driven flow (free convection). It is the ratio of buoyancy forces to the product of thermal and momentum diffusivities. It is also equivalent to the product of the Grashof number and the Prandtl number Ra = GrPr. The critical Rayleigh number (Ra_c) can be determined using linear stability theory. When the Rayleigh number is below the critical value for a fluid, heat transfer is primarily in the form of conduction, when it exceeds the critical value, heat transfer is primarily in the form of convection.

$$\text{Ra} = \frac{g\beta\Delta T H^3}{\upsilon\alpha} \qquad (2.17)$$

(viii) Dimensionless Porous parameter, Pm

It is defined as the ratio of boundary layer thickness to the square root of permeability.

$$\text{Pm} = \delta/\sqrt{k} \qquad (2.18)$$

Where δ - thickness of the boundary layer and k – permeability,. However, in the presence of variable porosity situation it is taken as

$$Pm = \frac{x\sqrt{150}}{\varepsilon_o d_p} \qquad (2.19)$$

2.5 Nomenclature

In order not to depart too drastically from the convention, as far as possible the normally employed symbols are used in this dissertation.

List of commonly used symbols in this dissertation are:

a - aspect ratio (H/L)

C_p – specific heat capacity at constant pressure ($Jkg^{-1}K^{-1}$)

d_p- particle diameter (m)

g – acceleration due to gravity (ms^{-2})

h – heat transfer coefficient ($Wm^{-2}K^{-1}$)

k – constant permeability (m)

H – v ertical characteristic length (m)

K – variable permeability of the porous medium[= $\frac{\varepsilon^3 d_p^2}{180(1-\varepsilon)^2}$] (m^2)

L – horizontal characteristic length (m)

Nu- Nusselt number

p – pressure (Nm^{-2})

Pe- Peclet number (R_eP_r)

Pm-Porous parameter

Pr- Prandtl number (υ/α)

q – total heat flux (Wm^{-2})

q_c – convective heat flux (Wm^{-2})

Re- Reynolds number

T – temperature (K)

u – velocity in the x-direction (ms^{-1})

U – velocity of the free stream (ms^{-1})

v – velocity in the y- direction (ms^{-1})

V – Darcian velocity vector (ms^{-1})

x – horizontal co ordinate (m)

y – vertical co ordinate (m)

Greek Symbols

α – thermal diffusivity [$= \lambda/\rho c_p$] (m^2s^{-1})

β - thermal expansion coefficient (K^{-1})

δ – hydrodynamic boundary layer thickness (m)

ε_o – mean porosity of the porous medium

λ_e – effective thermal conductivity [$= \varepsilon(\eta)\lambda_f + (1- \varepsilon(\eta)\lambda_s$] ($Wm^{-1}K^{-1}$)

λ_f – thermal conductivity of fluid ($Wm^{-1}K^{-1}$)

λ_s – thermal conductivity of solid ($Wm^{-1}K^{-1}$)

μ - dynamic viscosity of the fluid ($kgm^{-1}s^{-1}$)

υ – kinematic viscosity of the fluid (m^2s^{-1})

θ- dimensionless temperature

ρ – density of the fluid (kgm^{-3})

Subscripts

$_h$ – condition at the hot plate

$_c$ – condition at the cold plate

$_w$ – condition at the wall

$_o$ – condition at the entrance of the channel

∞ - condition at the edge of the boundary layer or free stream

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CHAPTER 3

AN EXPERIMENTAL STUDY OF NATURAL CONVECTION IN POROUS MEDIA HEATED FROM BELOW

ABSTRACT: The present experimental study is carried out using a rectangular cavity packed with porous medium such as iron balls or glass balls of different diameters and hence different porosity. The lower plate is maintained at a uniform temperature T_h, which is higher than the temperature T_c of upper plate. The experiment is performed for both closed and open systems in vertical as well as horizontal orientation of the test box. When (T_h - T_c) is increased beyond a certain value, convective motion takes place in the fluid. The results of the experiment indicate that a porous medium can transport more energy than a saturated fluid alone if the porous matrix is highly permeable and thermal conductivity of solid particles is greater than that of fluid. This experiment also shows that convective effect is more in open system i.e., free boundary at the top. It is also observed that the time taken to reach the thermal steady state is less in the horizontal orientation as compared to the vertical orientation. In this study, the relationship between Rayleigh and Nusselt numbers has been investigated in addition to the expected temperature profile. It is found that the magnitude of temperature for iron balls is about 10% more as compared to glass balls and about 18% higher for vertical orientation as compared to horizontal orientation.

3.1 Introduction

Natural convection occurs as a result of density inversion caused by either the thermal expansion of a fluid or the concentration gradients within the fluid system. The phenomenon of thermal convection through a porous layer has been observed so much in the natural world. For example; dispersion of heat through the porous layer composed of volcanic debris by natural convective movement of hot water under the ground, heat transmission from the ground (high temperature) to the atmosphere (low temperature) through a snow layer by natural convection of moist air, ground water pollution transport, nuclear waste repositors, solar collectors and petroleum reservoirs [1-2]. One of the simplest practical applications of the heat transfer theory is natural convection of liquids in vessels. Natural convection on very large scale occurs both in the atmosphere and within the earth. The phenomenon has long been of interest to geophysicists, astrophysicists and meteorologists. In petroleum reservoirs, natural convection occur as a consequence of geothermal gradients, thermal stimulation of a reservoir by steam or segregated forward combustion [3]. In all such situations it is highly beneficial to be able to predict the onset of natural convection within the reservoir and the contribution of the convective currents to the transfer of heat from one region to the other. Natural convection heat transfer has been a reliable, cost effective cooling method for the fast-growing electronic industry where hundreds of thermal connection modules are accommodated on a small base. As the density of these heat producing module increases day by day for more compactness, the heat released should be transferred from the surface not only to protect the unit from heat but also for longer life. Natural convection can also take place in porous medium saturated with a fluid when the medium is heated from below. In recent years, scientists have focused their attention on thermal insulation and heat storage using porous media. Low density insulating materials [4] are being used increasingly in consumer appliances, clothing, homes, automobiles, aircraft and industrial process equipment because of savings both space and weight. Usually more than 95% of their volume is occupied by gas. Typical solid materials now being employed in these insulations are polysterene, polyurethane, wood fiber and glass. Other investigators [5-6] extended the problem to compact heat exchanger experiments, gas turbines regenerator design and absorption in granular solids. In developing a porous heat storage of supply such as solar or geothermal energy it is necessary to obtain information about the transient characteristics of heat transfer in the porous layer. The purpose of the present study is to examine experimentally the transient and steady state characteristics of convective heat transfer in the presence as well as in the absence of porous medium. Though this problem has been done by several authors [7-8], no simple and low cost experiment has appeared in the literature. The present work highlights the comparison between vertical and horizontal orientations of the test section. The analysis is based on a one dimensional treatment with uniform bulk fluid velocities and temperatures at any cross-section normal to the direction of the flow. Heat transfer due to radiation and axial conduction are neglected and internal heat generation and chemical and/or nuclear reactions are assumed to be nonexistent.

3.2 Theory

It is known that the free or natural convection takes place in the still fluid due to its heating by the contact with the hot body. If a layer of fluid is heated from below, then the density of the bottom layer becomes lighter than the top. This results in variation of temperature, density and pressure between the bottom and the top plane. Therefore uniform linear gradient of density and temperature will be established. For sufficiently small values of the temperature difference (T_h - T_c) between the lower and upper plates, the viscous force overcomes the buoyancy force and the fluid remains motionless. Suppose the temperature difference (T_h - T_c) is increased beyond a certain value the buoyancy force overcomes the viscous force. Then the top heavy state becomes unstable and convective motion starts. The first intensive experiment was carried out by Benard in 1900. He observed the appearance of hexagonal cells when the instability occurs in the form of convection and it was Rayleigh who developed the theory for the instability in 1916. As a matter of fact, the gravity pulls the cooler denser fluid from the top to the bottom but the buoyancy force pushes the lighter fluid upwards. According to Rayleigh, the gravitational force between the fluid layer is opposed by the viscous damping force of the fluid. The balance of these two forces is expressed by a non-dimensional parameter called the Rayleigh number (R_a), the Rayleigh number for the case of fluid medium is

$$R_{af} = \frac{g\beta(T_h - T_c)H^3}{\upsilon\alpha}$$

Here, T_h –Temperature of hot wall, T_c –Temperature of the cold wall, H–height of the cavity, $\beta = 1/T_c$ – Coefficient of thermal expansion, υ –Kinematic viscosity,

g –acceleration due to gravity,

α (= $\lambda_e / \rho C_p$) – Thermal diffusivity, ρ – Density of fluid, C_p – Specific heat capacity of fluid at constant pressure, λ_e – Thermal conductivity of the mixture of fluid and solid or effective thermal conductivity.

$$\lambda_e = \varepsilon\lambda_f + (1 - \varepsilon)\,\lambda_s,$$

Where, ε – porosity, λ_f – thermal conductivity of fluid and λ_s – thermal conductivity of solid.

Nevertheles, the modified Rayleigh number (R_a) in the case of a porous medium is

$$R_{as} = \frac{g\beta(T_h - T_c)HK}{\upsilon\alpha}$$

Where, K is the permeability of the porous medium and it is given by

$$K = \frac{\varepsilon^3 d_p^2}{180\,(1 - \varepsilon)^2}$$

One should note that as R_a increases, the gravitational force becomes more dominant. At a critical Rayleigh number about 1700, the instability sets in and convection cells appear. The critical Rayleigh number can be obtained analytically for a number of different boundary conditions by doing a perturbation analysis on the linearized equations in stable state. In the case of a rigid boundary at the bottom and a free boundary at the top (without a lid) the critical Rayleigh number comes out as 1100. The other important parameters, which deal with natural convection are;

G_r–Grashof number, R_e–Reynolds number, P_r–Prandtl number and Nu-Nusselt number.

$$G_r = \frac{\rho U^2 \times \rho g \beta (T_h - T_c) H^3}{(\mu U)^2} = \frac{g \beta (T_h - T_c) H^3}{\upsilon^2}$$

$$R_e = \frac{UH}{\upsilon}, \quad P_r = \frac{\upsilon}{\alpha}, \qquad Nu = \frac{h \Delta T}{\lambda \Delta T / H} = \frac{hH}{\lambda}$$

If $\frac{G_r}{R_e^2} >> 1$, then natural convection occurs. In natural convection, flow velocities are produced by the buoyancy forces only, hence there are no externally induced flow velocities. As a result, the Nusselt number does not depend on the Reynolds number. $R_a = G_r P_r = G_r$ (since $P_r \approx 1$ for gases). Nu = f (G_r) = f (R_a). Infact, most free convective flows arise from instabilities created by heated fluid rising past and cooler fluid must descend. It can be observed that Nu remains equal to 1 below the critical Rayleigh number, but increases above unity with the onset of convection. For values of R_a not too much in excess of the critical value, convection always occurs in a fairly regular pattern. The individual elements in such a pattern are known as cells and the flow is called cellular. The geometry of the pattern can take a variety of forms such as rolls, triangular, rectangular, hexagonal and square cells with increase in R_a.

3.3 Experimental apparatus and Procedure

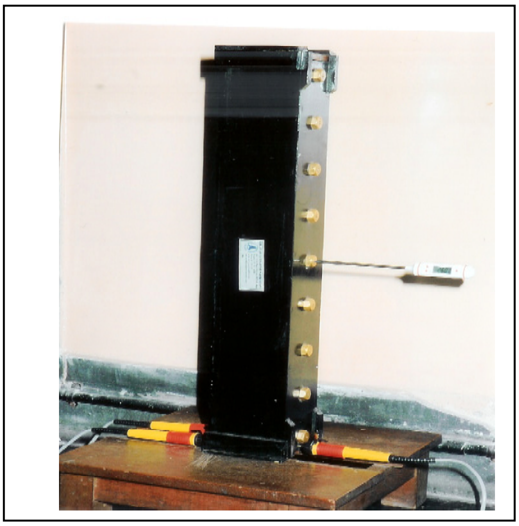

Fig. 1.

The experiment is performed by using a rectangular wooden box of width 95 mm, depth 99 mm and height 628 mm. But the cross sectional area of the test section is 85 mm x 89 mm. Fig. 1 shows the Experimental Apparatus.The main parts of the apparatus consists of heating and cooling parts along with porous media packed inside the box. Surface temperature of the hot wall (copper plate 10 mm in thickness) is maintained by using three strip heaters (soldering iron), each having a maximum capacity of 25 W. Actually two soldering rods are inserted on one side of the plate almost at its ends and on the opposite side one more soldering (third one) is inserted into the middle of the plate. Thus it is ensured that the bottom plate is uniformly heated and maintained at T_h , and the surface temperature of the cold wall T_c (copper plate 10 mm in thickness) is kept at room temperature. 9 holes are made on one of the vertical walls of the box and each hole is fitted with a conical shaped cap to introduce the digital thermometer. The holes/thermometers are located at 1, 8.3, 15.3, 22.7, 29.6, 36.9, 44.2, 51.4 and 58.5 cm from the bottom hot plate. It is taken care that no air enter into the box or leave from the box. That is the rectangular box is made air tight with bottom and the top plates isothermal (but at different temperatures) and the side walls adiabatic.

Measurements are made for three solid particles, glass balls of diameters 15 mm and 10.85 mm and steel balls 9.5 mm respectively with air as the fluid. In this work, the porosity is determined by measuring the volume ratio of filled water in the packed container to the empty container. It is obtained 0.415 and 0.419 for glass balls and 0.444 for iron balls. However, for spherical solid particles, the value of ε is almost unity near the wall surface due to one particle having only one contact point with the wall surface. For a packed bed as reported by Yagi et al [9], the porosity distribution from the wall surface to a distance of radius of solid particle in the direction of core portion of porous layer is varied from 1 to 0.4 and that inside of porous layer becomes constant.

In this study, the values of some similarity parameters are:

R_{af} (fluid) = 3.47×10^{7}, R_{as} (glass balls) = 1.21×10^{5}, R_{as} (iron balls) = 2.14×10^{3},

K (glass balls) = 2.34×10^{-5}, K (iron balls) = 1.42×10^{-7}.This shows that the values of R_a are very much smaller than the corresponding values for fluid alone. The experiment is conducted in a room, where the thermal disturbance from the environment is prevented. This is because the readings are likely to be affected by any cross flow of breeze (all the doors and windows of the room are closed and no fans are allowed to run). Temperature is measured at regular intervals of time till the steady state is reached. In fact, to reach the steady state, it takes about 4 hours. The experiment is also done by keeping the test section horizontal. In the case of a cavity a geometrical parameter, aspect ratio needs to be considered. When the test section is vertically oriented, aspect ratio H/W=6.38 and when it is horizontally oriented, aspect ratio W/H=0.156.The measured values for horizontal orientation is entirely different from vertical orientation. Further, the time taken to reach the steady state in horizontal orientation is less than that of the vertical orientation.

3.4 Experimental results and Discussion

To gain a perspective of the physics of heat transfer by natural convection, the experimental arrangement is made in such a way that the lower boundary at y=0 is maintained at a temperature T_h and the upper boundary y=H is at T_c, with $T_h > T_c$. But, T_c is dependent on the medium in the enclosure. The porous medium has scalar permeability K and is saturated with fluid of density ρ and viscosity μ. We now consider the test section as described above and shown in Fig.1. The height of the vertical channel is H, while separation is W. It is noted that the temperature near the heated wall increases because of the heating effect from the wall(bottom). At the other end, the temperature near the cold wall(top) decreases because of the cooling effect from the surroundings.

Figure 2 shows that the typical non-dimensional temperature distribution (θ) for the aspect ratio of H/W=6.38 and the width of cavity W=95 mm. In what follows is the temperature distribution in the case of vertical section filled with iron balls or glass balls as porous medium and absence of porous medium (air alone). The temperature distribution for iron balls and glass balls are similar, but the magnitude of temperature for iron balls is more, about 10% at 0.137(y= 8.3 cm) as compared to glass balls. However, in the absence of porous medium, the magnitude of temperature is more by about 7% as compared to glass after 8.3 cm. The reason for more temperature distribution in the presence of iron balls near the hot wall indicates that the intensity of the convection is weak due to the large thermal conductivity.

Figure 3 shows the temperature distribution in both the vertical and horizontal orientation of the test section. The magnitude of the temperature in the presence of glass balls in the horizontal orientation is of little attention. However, in the absence of porous medium, there is considerable difference between the vertical and horizontal orientations. Infact, magnitude of the temperature in the vertical orientation is about 18% higher (at 0.016 or y=1 cm) than the horizontal orientation. Nevertheless, it keeps decreasing as one approaches the cold wall.

Figure 4 exhibits the temperature profile (θ) for the aspect ratio 6.38 and glass balls of different diameters. It is found that there is no much difference in magnitude of temperature even though the diameters are different.

TABLE 1. Percentage variation of θ for air and Iron balls

y/H	0.016	0.137	0.252	0.374	0.488	0.609	0.729	.848	0.965
Air	2.85	1.08	-0.21	-0.12	-0.02	0.23	0.4	0.83	1.53
Iron balls	1.62	0.86	1.88	0.73	0.17	0.04	-0.05	0.0	0.0

Table 1 shows the variation of dimensionless temperature θ, when the upper boundary is changed from a rigid-lid to a constant pressure surface. Removal of the lid leads to enhanced convective motion because the hot plume is free to leave the system and its width narrower. Where as, in the confined case the cold plume emerges from a thermal boundary layer along the top surface. As such, descending fluid is generally warmer than in the case of a permeable top, where all fluid entering the porous medium is at the temperature of the cold upper surface.

TABLE 2. Nusselt number and Rayleigh number

R_a	Nu
3.47×10^7	749.6
1.22×10^5	51.0
2.14×10^3	7.86

The results of this experiment is compiled in terms of the usual dimensionless groups, namely, the Nusselt number and the Rayleigh number. Table 2 shows the relationship between the steady state Nusselt number and the Rayleigh number for the systems investigated.

3.5 Conclusions

The experiment is carried out by the rectangular cavity packed with/without porous media, whose opposing horizontal walls have different but uniform temperatures and the other walls are adiabatic. The following conclusions are obtained.

1. An increase of the natural convection in the porous layer causes a shorter time to reach a steady state and thermal homogeneous in the porous medium

2. A porous medium can transport more energy than a saturated fluid alone if the permeability and thermal conductivity of solid particles is greater than that of fluid

3. The magnitude of the temperature in the vertical orientation is about 18% higher than that of the horizontal orientation.

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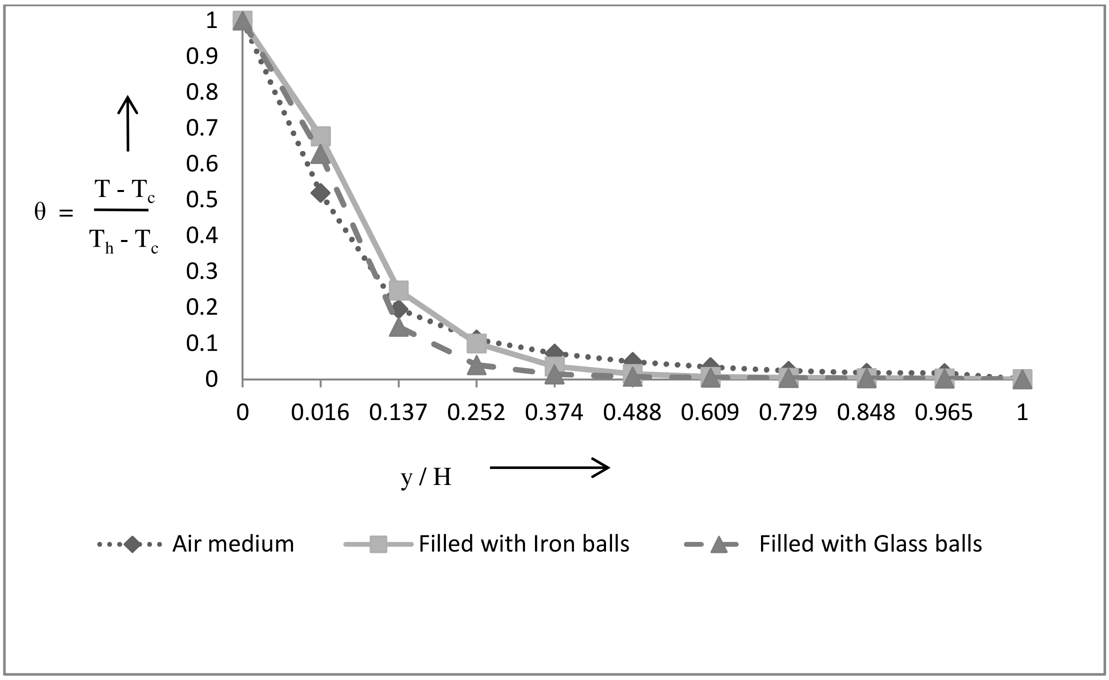

Fig. 2. Temperature distribution in a vertical porous layer for W = 95 mm, H/W = 6.38

$T_h = 260^0$C, $T_c = 30^0$ C (for air medium), $T_c = 25^0$ C (for glass and iron balls)

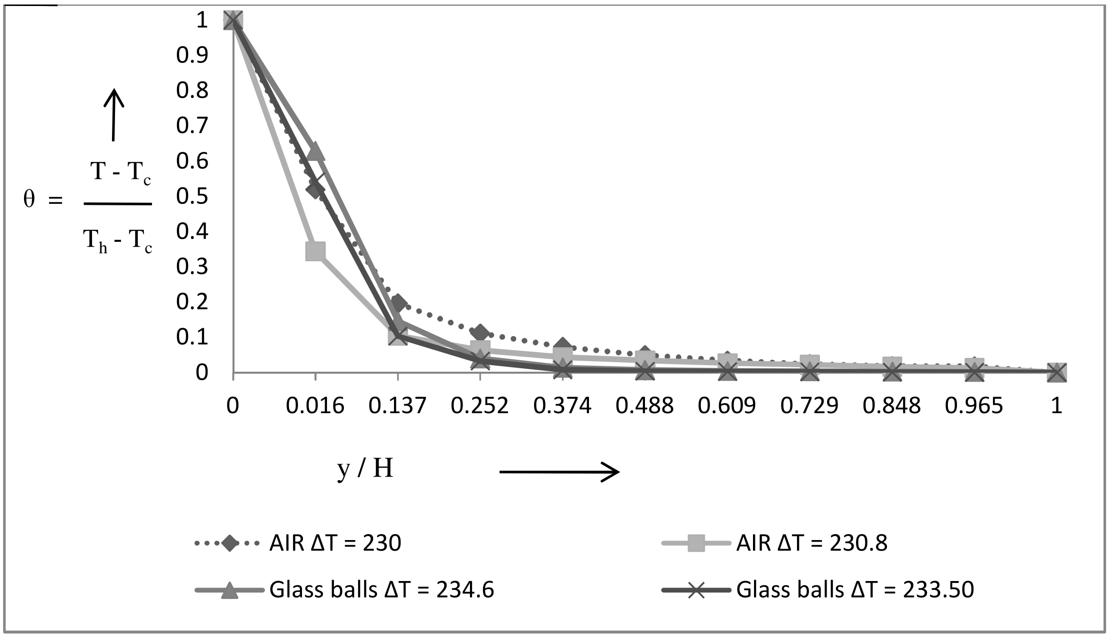

Fig. 3. Temperature distribution in the vertical and Horizontal porous layer for W = 95 mm, W/H = 0.156 ,$T_h = 260^0$C, T_c (Vertical, air) = 30^0 C, T_c (Horizontal, air) = 29.2^0 C, T_c (Vertical, glass balls) = 25^0 C, T_c (Horizontal, glass balls) = 26.5^0 C

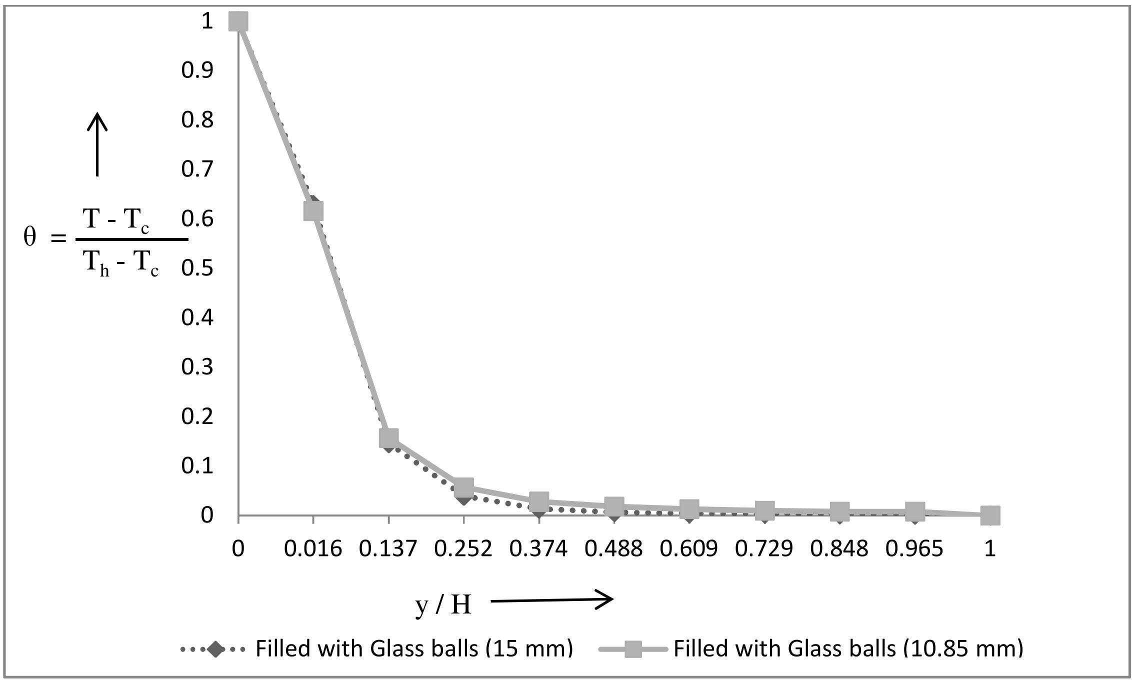

Fig. 4. Temperature distribution in a vertical porous layer for W = 95 mm, H/W = 6.38

$T_h = 260^0C$, $T_c = 25^0$ C

CHAPTER 4

EFFECT OF RADIATION ON BOUNDARY LAYER FLOW OF AN EMITTING, ABSORBING AND SCATTERING GAS IN A VARIABLE POROSITY MEDIUM

ABSTRACT: The heat transfer by convection and radiation in boundary layer flow over a flat plate embedded in a saturated porous medium is studied. The effects of variable porosity and thermal conductivity of the medium and also the emission, absorption and scattering of radiation are taken into account. The comparative study has been made for three different situations, namely a) variable porosity b) constant porosity and c) absence of porosity. In carrying out the solution, as a first step the temperature profile within the radiation layer is determined. From this, the temperature at the outer edge of the boundary layer is obtained. The momentum and energy equations are coupled and they are solved simultaneously by Runge-Kutta Gill method. The results of the analyses show that, in the cases of variable porosity and absence of porosity media the velocity profiles possess very small curvature, whereas, in the case of constant porosity situation the velocity profile is almost zero up to a certain distance and then increases. Nevertheless, it reaches unity asymptotically in all the three cases. The temperature profile becomes linear as the value of b (ratio of thermal conductivity of solid to fluid $-\lambda_s/\lambda_f$) increases. Another important result of the analyses is that the rise in temperature in variable porosity medium is about 25% more as compared to the case of constant porosity medium.

Nomenclature

b ratio of thermal conductivity of solid to fluid=λ_s/λ_f

c, d constants, defined in Eq (2)

c_p specific heat capacity at constant pressure ($Jkg^{-1}K^{-1}$)

d_p particle diameter (m)

e radiation energy = σT^4 (Wm^{-2})

$G(\theta_w)$ defined in Eq (32)

$G'(\theta_w)$ defined in Eq (33)

$H_0(\eta)$ defined in Eq (25)

H defined in Eq (36)

$k(y)$ variable permeability of the porous medium (Eq.2) (m^2)

K_a, K_s absorption and scattering coefficient defined in Eq (8)

p_m porous parameter, $P_m^2 = \frac{150\,x^2}{\varepsilon_0^2 d_p^2}$ Eq (18)

q_r radiation flux defined in Eq (6)

R_e Reynolds number = $\frac{U_\infty \chi}{\upsilon}$ Eq (18)

T temperature (K)

u, v velocity components along the x-and y-axes respectively (ms^{-1})

x co-ordinate along the plate from the forward edge (m)

y co-ordinate perpendicular to the plate (m)

Greek symbols

δ hydrodynamic boundary layer thickness = $\sqrt{\frac{\nu x}{U_\infty}}$ (m)

$\varepsilon(y)$ variable porosity, defined by Eq (2)

ε_o mean porosity of the porous medium

ϵ emissivity Eq (4)

$f'(\eta)$ dimensionless velocity profile = $\frac{u}{U_\infty}$

Φ non-dimensional radiation flux= $\frac{q_w}{\sigma T_w^4}$

η dimensionless variable = y/δ

$\psi(x, y)$ stream function =$\sqrt{\nu x U_\infty}\, f(\eta)$

λ_e effective thermal conductivity = $\varepsilon(y)\lambda_f + [1 - \varepsilon(y)]\lambda_s$ Eq (4)

λ_f , λ_s thermal conductivity of fluid and solid ($Wm^{-1}K^{-1}$)

Λ dimensionless conductive heat flux = $-\frac{\lambda_e}{\delta\sigma T_w^3}\frac{d\theta}{d\eta}$ Eq (35)

Λ_1 constant defined Eq (25)

μ dynamic viscosity ($kgm^{-1}s^{-1}$)

ν kinematic viscosity (m^2s^{-1})

θ dimensionless temperature (K)

θ_0, θ_1, θ_2 – temperature distribution defined in Equations (22-24)

ρ density of the fluid (kgm^{-3})

σ stefan constant ($Wm^{-2}K^{-4}$)

τ optical depth defined in Eq (6)

τ_o optical thickness = $\frac{(K_a + K_s)\chi}{\sqrt{R_e}}$ defined in Eq (30)

ξ non-dimensional number $= \frac{2x\sigma(K_a+K_s)}{\rho C p U_\infty} T_\infty^3$ *defined in Eq (14)*

Subscripts

w condition at the wall

∞ condition at the free stream

4.1 Introduction

The study of simultaneous radiative and convective heat transfer problems in porous media are of considerable practical importance in many engineering applications. Most of the studies in porous media carried out are based on the Darcy flow model, which in turn is based on the assumption of creeping flow through an infinitely extended uniform medium[1] such as fixed bed catalytic reactors, packed bed heat exchangers, drying, chemical reaction engineering, and metal processing. The constant porosity assumption does not hold good because of the influence of an impermeable boundary. The permeability and porosity measurements by Roblee et al [2] and Benenati and Brosilow [3] show that, due to the packing of particles porosity cannot be taken uniform but has a maximum value at the wall and a minimum value away from the wall. Hence, one has to incorporate the variation of porosity to study the heat transfer rate accurately. To account for the effects of the solid boundary, inertia forces, and variable porosity on fluid flow and heat transfer rate through porous media, Brinkman's extension of Darcy's law should be used [4]. Chandrasekhara and Vortmeyer [5] and Vafai [6] have incorporated the variable permeability to study the flow past and through a porous medium and have shown that the variation of porosity and permeability have greater influence on velocity distribution and on heat transfer. Earlier publications on heat transfer in a variable porosity medium have considered convection and conduction only [7] and have neglected the effect of thermal radiation. It has been found that even under some of the most unexpected situations such as in fur [8] and building insulations, radiation heat transfer could account for a non-negligible amount of the total heat transfer. Tong et al [9] have reported in their work that the radiant heat transfer in light weight fibrous insulations accounts for as much as 30% of the total heat transfer even under moderate temperature (300-400 K).The process of heat transfer in high temperature systems become increasingly important with the improvement on thermal efficiency of plant. Heat removal from high temperature gas reactor is one such process. It is considered that as a practical application the fibrous materials or the sintered materials with very high porosity are installed in duct as pieces for absorbing radiant energy from the wall [10-13]. As a matter of fact in fluidized bed systems, convection and radiation are the important mechanisms of energy transfer as indicated by experimental studies of Goshayeshi et al [14]. In the works quoted above, the authors have not considered the effect of variable porosity as well as variable conductivity of the medium. Motivation of this paper is to study the role of thermal radiation in the overall heat transfer problem in the presence of property variation namely permeability and conductivity. The volume averaged equation proposed by Vafai [6] is used and correlation between porosity and permeability is brought through Kozeny-Blake expression. In a closely packed system the scattering effect is neglected [15-19]. However, in a sparsely packed system the scattering effect cannot be neglected and hence it is incorporated by the extinction coefficient ($K_a + K_s$). As reported by Cess [20], if there are no suspended particles or droplets within the gas, the only type of scattering that will occur is Rayleigh scattering. On the other hand, in the presence of suspended particles or droplets, gross scattering will result and this mode of scattering is independent of wavelength and will generally be significant. Radiation combined with other modes of heat transfer is highly nonlinear integro-differential equation whose exact analytical solution is nearly impossible. Hence an efficient tool to deal with multi dimensional radiative heat transfer is in strong demand. Thus the problem involves a set of coupled equations with variable coefficients, which are solved by Runge-kutta Gill method in conjunction with Newton-Raphson iterative scheme.

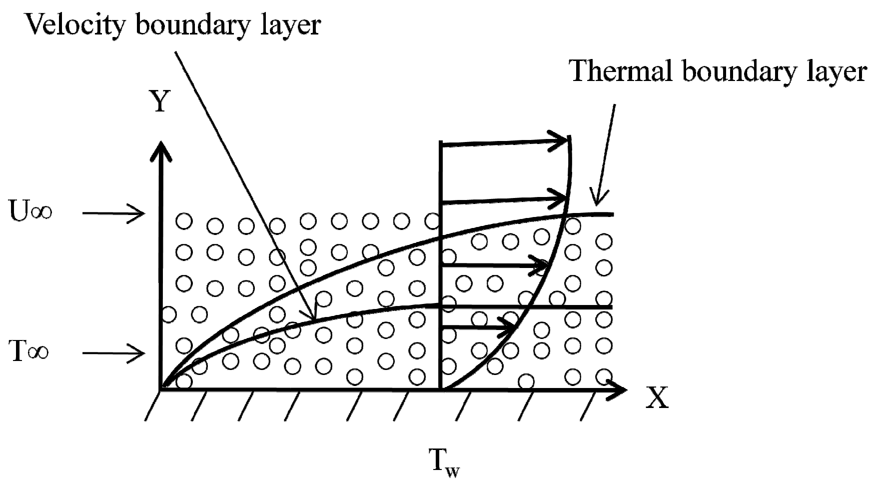

Fig. 1 Geometry and Physical system

4.2 Mathematical formulation and Boundary conditions

The physical model and co-ordinate system are illustrated in Fig.1. It consists of a steady laminar flow of gray fluid flowing past a flat plate with negligible viscous dissipation and surface temperature of the plate is taken to be uniform.

The foregoing continuity, momentum, and energy equations for a radiating fluid are similar to those for a non-radiating fluid except for the radiative heat flux term $-\partial q_r/\partial y$ appearing in the energy equation.

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y} = 0 \qquad (1)$$

$$\rho\left(u\frac{\partial u}{\partial x}+ v\frac{\partial u}{\partial y}\right)=\frac{\mu\, \partial^2 u}{\varepsilon(y)\partial y^2} - \frac{\mu}{k(y)}u \qquad (2)$$

$$where,\ \varepsilon(y) = \varepsilon_0\left(1+ce^{-\frac{yd}{d_p}}\right) and\ k(y) = \frac{\varepsilon(y)^3 d_p^2}{150[1-\varepsilon(y)]^2}$$

Here $\varepsilon(y)$ and k(y) are the expressions for variable porosity and permeability (Kozeny-Blake expression) respectively. ε_0 is the mean porosity and its value is chosen as 0.4, c and d are empirical constants which depend on the packing of spheres and d_p is the particle diameter.

$$\left(u\frac{\partial T}{\partial x}+ v\frac{\partial T}{\partial y}\right) = \lambda_e\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{3}$$

Where q_r denotes the radiation flux in the y-direction, $-\frac{\partial q_r}{\partial y}$ is the net radiation transfer to the fluid per unit volume and λ_e represents the effective thermal conductivity of the medium and it is given as [6]

$$\lambda_e = \varepsilon(y)\lambda_f + [1- \varepsilon(y)]\lambda_s \tag{4}$$

The boundary conditions are taken as

$$at \qquad y = 0,\ u = v = 0,\ T = T_w \tag{5a}$$

$$as \qquad y \to \infty,\ u \to U_\infty,\ v = 0,\quad T \to T_\infty \tag{5b}$$

4.3 ANALYSIS

In the analyses of radiation effects upon the boundary layer flow, Cess [20] has introduced a model, according to which conduction is restricted within the radiating fluid to a thin region adjacent to the plate surface. This conventional boundary layer is optically thin, τ_0<<1. However, the optically thin boundary layer represents only a portion of the entire temperature field, and consequently it is necessary to consider not only the boundary layer but also the adjacent radiation layer. In carrying out the solution, firstly the temperature profile within the radiation layer is determined. From this, the temperature at the outer edge of the boundary layer is obtained [21]. The monochromatic radiant flux for a gray fluid in terms of an optical depth τ may be expressed as [22-24]

$$q_r = 2\epsilon_w e_w E_3(\tau) + 4\,(1-\epsilon_w)E_3(\tau)\int_0^\infty e(x,\tau)E_2(\tau)\,d\tau$$

$$+ 2\int_0^\tau e(x,t)E_2(\tau - t)\,dt - \int_\tau^{\tau_0} e(x,t)E_2(-\tau)\,dt \tag{6}$$

where $\tau = \int_0^y (K_a + K_s)\,dy$ *is optical depth and* $E_n(t) = \int_0^1 \mu^{n-2}\, e^{-t/\mu} d\mu$

On differentiation Eq. (6), it yields

$$-\frac{\partial q_r}{\partial \tau} = 2\epsilon_w e_w E_2(\tau) + 4(1-\epsilon_w)E_2(\tau)\int_0^{\infty} e(x,\tau)E_2(\tau)\,d\tau$$

$$+2\int_0^{\tau_0} e(x,t)E_1(|\tau t|) - 4e(x,\tau) \qquad (7)$$

4.3.1 Radiation layer. Since the fluid is assumed to have uniform velocity everywhere; u = U_∞ and v=0, upon neglecting the conduction within the radiation layer, the energy Eq. (3) reduces to

$$U_\infty \frac{\partial T}{\partial x} = -\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \quad or \quad U_\infty \frac{\partial T}{\partial x} = -\frac{(K_a+K_s)}{\rho c_p}\frac{\partial q_r}{\partial \tau} \qquad (8)$$

Now from Eqs (7) and (8) one obtains the energy equation as

$$\frac{\partial T}{\partial x} = \frac{(K_a+K_s)}{\rho c_p}\left[2\epsilon_w e_w \mathrm{E}_2(\tau) + 2\int_0^{\tau_0} e(x,t)\,E_1(|\tau - t|)\mathrm{dt} - 4e(x,\tau)\right] + \cdots \quad (9)$$

It is considered that the free stream temperature of the fluid flowing over an isothermal flat plate embedded in a saturated porous medium is T_∞ at x=0, and the plate surface is black, therefore ‘e’ becomes e_∞. Upon integrating Eq. (9), and by taking the first approximation as T = T_∞ one obtains the following expression for T [20]

$$T = T_\infty + \frac{2(K_a+K_s)}{\rho c_p U_\infty}\epsilon_w x[e_w - e_\infty]\,E_2(\tau) + \ldots. \qquad (10)$$

This expression (Eq.10) is the boundary condition at the outer edge of the boundary layer. In arriving at this equation it is assumed that $\int_0^{\infty} E_2(\tau)\,d\tau = 1/2$ and $E_2(\tau)=1+O(\tau)\approx 1$

4.3.2 Boundary layer. Again taking as a first approximation e = e_∞, for τ_0 <<1 (i.e., within the boundary layer $\tau > \tau_\delta$) Eq.(7) reduces to

$$\frac{\partial q_r}{\partial \tau} = -[2\epsilon_w(e_w - e_\infty) + 4(e_\infty - e)] \tag{11}$$

Using Eq. (11) in Eq. (3), the energy equation takes the form

$$\rho c_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \lambda_e \frac{\partial^2 T}{\partial y^2} + 2\sigma(K_a + K_s)[\epsilon_w(T_w^4 - T_\infty^4) + 2(T_\infty^4 - T^4)] \tag{12}$$

The boundary conditions for isothermal plate surface specification together with Eq.(10) are:

$$at\ y = 0,\ T = T_w \tag{13a}$$

$$as\ y \to \infty,\ T = T_\infty + \epsilon_w(T_w^4 - T_\infty^4)\left[\frac{2(K_a+K_s)x}{\rho c_p U_\infty}\right]\mathrm{E}_2(\tau)+.. \tag{13b}$$

The basic equations are made non-dimensional through the introduction of the following similarity variables

$$\eta = y/\delta;\ f^{'}(\eta) = u/U_\infty\ ;\ \theta = T/T_\infty;\ \theta_w = T_w/T_\infty;\ R_e = U_\infty x/v\ ;$$

$$\xi = \frac{2x\sigma(K_a + K_s)T_\infty^3}{\rho c_p U_\infty} \tag{14}$$

Where $\delta = \sqrt{\frac{\upsilon x}{U_\infty}}$ boundary layer thickness and ξ- ratio of radiative flux to the incoming enthalpy flux and it also involves the absorption and scattering of the medium. It should be noted that the boundary layer 'y' varies from 0 at the wall to δ at the boundary limit. Thus δ is not a function of x but can be determined at x=L.

Continuity equation is satisfied by introducing a stream function ψ(x, y) such that

$$u = \frac{\partial \psi}{\partial y}\ , \qquad v = -\frac{\partial \psi}{\partial x} \tag{15}$$

The steam function in this case is f the form

$$\psi(x,y) = \sqrt{\nu x U_\infty}\ f\ (\eta) \qquad (16)$$

Where f(η) denotes the dimensionless stream function. Then the velocity components become

$$u = U_\infty f' \qquad v = \frac{1}{2}\sqrt{\frac{U_\infty\,\nu}{x}}\ (\eta f' - f) \qquad (17)$$

where the prime denotes the differentiation with respect to η. Using the above notations momentum Eq.(2) takes the form

$$2f''' + \varepsilon_0[\,1 + ce^{-d\gamma\eta}]ff'' - \frac{2p_m^2\ \left[1-\varepsilon_o\ \{1+ce^{-d\gamma\eta}\}\right]^2}{R_e\left[\,1+ce^{-d\gamma\eta}\right]^2}f'\ = 0 \qquad (18)$$

$where\ p_m^2 = 150x^2/\varepsilon_0^2 d_p^2\ ,\ R_e = U_\infty x/v\ ,\gamma = \delta/d_p$, $d = x/d_p$

The value of c = 0 for constant porosity and absence of porous medium and c=1 for variable porosity. The constant d is based on the length of the flat plate and particle diameter.

For constant porosity situation Eq.(18) reduces to

$$2f''' + ff'' - \frac{2\rho_m^2}{R_e}f' = 0 \qquad (19)$$

The transformed boundary conditions are

$$at\ \ \eta\ =\ 0, f\ =\ f' = 0 \qquad (20a)$$

$$as\ \ \eta\ \rightarrow\ \infty\ ,\ f' =\ 1 \qquad (20b)$$

The solution of energy Eq. (13b) will now be assumed of the form [6]

$$\theta = [\,1 + (\theta_w - 1)\ \theta_0(\eta)] + \ (\ \theta_w^4 -\ 1\,)\ [\ \theta_1(\ \eta) +\ (\ \epsilon_w - 1\ )\ \theta_2(\eta\ )\ ]\xi + .. (21)$$

Substituting Eqs. (17) and (21) into Eq. (12) and collecting like powers of ξ, the ordinary differential equations describing θ_0, θ_1 and θ_2 are obtained.

$$\frac{\theta_0''(\eta)}{P_r}\Lambda_1 + \frac{f(\eta)}{2}\,\theta_0'(\eta) = 0 \tag{22}$$

$$\frac{\theta_1''(\eta)}{P_r}\Lambda_1 + \frac{f(\eta)}{2}\,\theta_1'(\eta) - f'(\eta)\theta_1(\eta) = -H_0(\eta) \tag{23}$$

$$\frac{\theta_2''(\eta)}{P_r}\Lambda_1 + \frac{f(\eta)}{2}\,\theta_2'(\eta) - f'(\eta)\theta_2(\eta) = -1 \tag{24}$$

Where $\Lambda_1 = \varepsilon_0[\,1 + ce^{-d\gamma\eta}] + \,b\,[\,1 - \,\varepsilon_0\{1 + ce^{-d\gamma\eta}\}]$

$$Ho\,(\eta) = \frac{1}{(\theta_w^4-1)}[\,1 + \,\theta_w^4 - \,2\{\,1 + (\,\theta_w - \,1)\theta_0\}^4] \tag{25}$$

The transformed boundary conditions using Equations (5) and (21) will now take the form

$$at\,\eta = 0,\ \theta_0(0) = 1,\ \theta_1(0) = 0,\ \theta_2(0) = 0 \tag{26a}$$

$$as\,\eta \to \infty,\ \theta_0(\infty) = 0,\ \theta_1(\infty) = 1,\ \theta_2(\infty) = 1 \tag{26b}$$

According to Cess [20] and Krishnameti and Ramakhandran [21] the function $\theta_0(\eta)$ is the temperature distribution for the case of negligible radiation interaction ($\xi = 0$) and the second bracketed term in Eq. (21) denotes the first order radiation effect on the temperature profile within the gas.

4.3.3 Wall heat flux. The net heat flux at the wall is of interest in most engineering applications. For a wall that is impermeable to flow, the net heat flux at the wall q_w is composed of the conductive and radiative heat fluxes and given as [23]

$$q_w = \,q_c + \,q_r = [\,-\lambda_e\frac{dT}{dy} + \,q_r]_{\,y\,=0} \tag{27a}$$

This can be expressed in terms of the dimensionless quantities as

$$\psi = \frac{q_w}{\sigma T_w^4} = \left[-\frac{2\tau_0}{P_r \xi}\frac{d\theta}{d\eta} + \Phi\right]_{\eta=0} \qquad (27b)$$

In evaluating the heat transfer between the plate surface and the medium, it is convenient to consider separately the radiative and convective transfers. To determine the radiation heat transfer at the plate surface, Eq. (6) with $\tau = 0$ gives

$$q_{rw} = \epsilon_w(e_w - e_\infty) - 2\epsilon_w \int_0^\infty (e - e_\infty)\, d\tau \qquad (28)$$

Eq. (28) can also be written as

$$\frac{q_{rw}}{\sigma T_w^4} = \epsilon_w(\theta_w^4 - 1)\left[1 - \frac{2\tau_0}{(\theta_w^4 - 1)}\int_0^\infty (\theta^4 - 1)\, d\tau\right] \qquad (29)$$

Upon substituting Eq. (21) into Eq. (29) one obtains

$$\frac{q_{rw}}{\sigma T_w^4} = \epsilon_w(\theta_w^4 - 1)\left[1 - \frac{2\tau_0}{(\theta_w^4 - 1)}\int_0^\infty \{[1 + (\theta_w - 1)\theta_0(\eta)]^4 - 1\}\, d\eta\right] \qquad (30)$$

The second term on the right side of Eq. (30) represents the first order correction and $\tau_0 = (K_a + K_s)\, x / \sqrt{R_e} = (K_a + K_s)\delta$, is a measure of optical thickness of the boundary layer, which is based on the characteristic dimension δ. Further, Eq. (30) can be modified as

$$\Phi = \epsilon_w(\theta_w^4 - 1)[1 - G(\theta_w)\tau_0] + \ldots \qquad (31)$$

Where $G(\theta_w)$ is given as

$$G(\theta_w) = \frac{2}{(\theta_w^4 - 1)}\int_0^\infty [\{1 + (\theta_w - 1)\theta_0(\eta)\}^4 - 1]\, d\eta \qquad (32)$$

It should be noted that the second term on the right side of Eq (31) represents the first order radiation effect. It depends only upon the optical thickness τ_0 and temperature ratio θ_w and not on the expansion parameter ξ for an isothermal plate of unit emissivity.

By differentiating Eq. (32) w.r.t η one obtains

$$G'(\theta_w) = \frac{2}{(\theta_w^4 - 1)}[\{1 + (\theta_w - 1)\theta_0(\eta)\}^4 - 1] \tag{33}$$

In order to solve the above equation, the boundary condition is taken as

$$G(\theta_w) = 0 \text{ at } \eta = 0 \tag{34}$$

The dimensionless conductive heat flux at the wall can be obtained as

$$\Lambda = -\frac{\lambda_e}{\delta\sigma T_w^3}\frac{d\theta}{d\eta} \tag{35}$$

The above equation can also be written as

$$\Lambda = -\frac{2\tau_0}{P_r\xi}\frac{d\theta}{d\eta} = -\frac{2\tau_0}{P_r\xi}[(\theta_w - 1)\theta_0'(0) + (\theta_w^4 - 1)H\xi] \tag{36}$$

Where $H = [\theta_1'(0) + (\epsilon_w - 1)\theta_2'(0)]$

Generally the convective heat transfer is expressed in terms of the Nusselt number

$$Nu = \frac{q_{cw}\,x}{\lambda_e(T_w - T_\infty)} = -\frac{x\theta'(\eta)}{(\theta_w - 1)\delta} \tag{37}$$

The convective heat transfer between the gas and the plate can be determined from equations (21) and (37)

$$\frac{Nu}{\sqrt{R_e}} = -\theta_0'(0) - \frac{(\theta_w^4 - 1)}{(\theta_w - 1)}H\xi \tag{38}$$

4.4 Solution Method

The momentum and the energy equations are independent of each other.

According to Krishnameti etal[21] when the momentum equation is solved, it yields a function f″(0), which is subsequently used for solving energy equation. In the present analyses, the momentum and energy equations (18) - (24) and (18) and (33) are coupled and they are solved simultaneously by Runge-Kutta-Gill method in conjunction with the Newton-Raphson iterative scheme. The advantage of the coupling in this problem is that the values of f (η) and f′(η) are automatically plugged in the energy equation. The general method of solution can be described as follows. Since f″(0) is the only unknown in equations (18) and (19), a rough estimate is made for f ″(0) for any specified value of η and then momentum equation is integrated. Solution of momentum equation yields function f″(0) which is subsequently used for solving Eqs(22) – (24) and then Eq (33). Computations are performed in double precision with 16000 steps for η i.e., η varying from 0 to 40 with constant step size $\Delta\eta$ = 0.0025. The convergence criterion employed herein required that the difference between the current and the previous iteration 10^{-6}. Further, the radiation transfer $G(\theta_w)$ is determined by taking the boundary condition zero, since $G(\theta_w)$ is an integral equation with 0 to ∞. On differentiation it yields a linear differential equation w.r.t η. The end point η = 40 is treated as infinity. In order to asses the validity of the solution, firstly the results are obtained in the absence of porous media. These results are in complete agreement with the results of Cess [20] and Krishnameti [21] Further, the values given of f, f ′ and f ″ are exactly matching with the values given in Chandrasekara and Nagaraju [18] in the absence of porous medium.

4.5 Results and Discussion

The problem is characterized by a large number of governing parameters and hence the numerical results are given below for the fixed values of P_r, P_m and R_e. In the present study the following typical values are used; porosity ε_o=0.4, particle diameter d_p=0.01and 0.02 m, and the free stream velocity U_∞=1ms^{-1}. With air as the reference fluid for a typical bed of length x=0.1 m, a local distance from the leading edge of the plate, the values of d, γ, R_e and P_m become 5, 0.01086, 70559 and 153 respectively. The mean porosity ε_0 far from the wall is taken as 0.4 for the variable and constant porosity situations. But, in the case of absence of porous medium ε_0 is taken as unity. Further, for the cases of constant porosity and absence of porous medium the value of c is taken as zero. For the free stream temperature T_∞ = 1000 K and wall temperature T_w = 500 K the value of ξ becomes approximately 0.01. The parameter ξ gives the relation ship between the radiative flux and the incoming enthalpy flux. The function $\theta_0(\eta)$ represents the temperature distribution for the case of negligible radiation interaction ($\xi = 0$), while $\theta_1(\eta)$ and $\theta_2(\eta)$ denote the first order radiation effects upon the temperature profile within the gas as indicated in Eq.(21).

The dimensionless velocity component $f'(\eta)$ is represented in Fig.2 for three different cases, namely variable porosity, constant porosity and absence of porous media. In the cases of variable porosity and the absence of porosity media the velocity profiles possess very small curvature, at the wall whereas, in the case of a constant porosity situation, the velocity is zero almost up to a certain distance from the wall. However, after a certain distance the velocity goes on increasing and approaches unity asymptotically. As expected the temperature is more in the presence of variable porosity medium compared to constant porosity medium. The rise in temperature is found to be about 25% more in comparison with the absence of porous medium.

Figure 3 exhibits the temperature profile in the presence of radiation ($\xi \approx 0.01$). It shows that the presence of radiation increases the temperature distribution. For a cold plate (Temperature ratio θ_w=0.5) the profile is concave downward in the limited value of $\eta(\approx 5)$, representing heat transfer from the medium to the wall. The cooling of the gas in the radiation wall layer reduces markedly the temperature at the outer edge of the boundary layer. As a result of this, the thermal processes occuring have little effect on the radiant flux density incident on the plate surface. The peak value in the negative direction occurs around η=2, and then the change of sign takes place between η=5 and 6. From η=6 onwards, the temperature increases. It can also be noticed that with increase in θ_w, the peak value in the negative direction decreases and for $\theta_w>2$, the temperature distribution becomes totally positive and increases with η and reaches its maximum value at η=8.

Figure 4 shows the temperature distribution in the presence as well as in absence of radiation. As would be expected, the temperature distribution decreases in the absence of radiation and increases in the presence of radiation.

Figure **5** shows that the temperature increases with increase **i**n b (= λ_s/λ_f). It is the ratio of thermal conductivities of solid to fluid, which has considerable influence on the flow and heat transfer characteristics. As an aid to understand the importance of transport phenomena there is a need to focus attention on the behavior of temperature with respect to conductivities of various solid fluid combinations constituting the porous bed. According to Vedhanayagam et al [23] as the value 'b' increases, the effective thermal diffusivity of the saturated porous medium close to the boundary layer decreases. This results in a steeper temperature gradient close to the wall and a slowly decaying temperature profile away from the wall. It is also noticed from Fig.5 that for b=1650, the temperature profile decreases linearly. In fact, the values of 'b' are chosen from the experimental data provided by Jaguaribe et al [25]

The conductive heat transfer between the gas and the medium is depicted in Fig.6, and the quantities are listed in Table 1. It may be seen from Fig. 6 that the emissivity of the medium has a relatively strong influence upon the conductive heat transfer. The decrease of ϵ_w results in increase in heat transfer, which can be explained as follows; the gas near the surface receives net radiation from the heated surface and gives up net radiation to the cooler free stream gas. Thus, a reduction in emissivity of the medium decreases the radiation heat transfer to this portion of the gas, and hence the conductive heat transfer increases. It may also be noted that from Table 1 that $G(\theta_w)$ is always positive. This may be explained from the fact that the gas within the boundary layer differs from the free stream temperature in the direction of T_w, such that radiation exchange between the portion of the gas and plate is reduced. The quantities $\theta_0'(0)$ and $\theta_1'(0)$ are also listed in Table1. It is to be noted that from Eq (31) the first order radiation term depends only upon the optical thickness

τ_o and the temperature ratio θ_w for an isothermal plate of unit emissivity.

In Fig.7 both cooling ($T_w < T_\infty$) and heating ($T_w > T_\infty$) cases are shown for the radiative flux (Φ). As can be seen from Table1 that $G(\theta_w)$ is always positive, such that the effect of the first order radiation term in Eq (31) is to reduce the heat transfer. If the plate surface is cooled ($T_w < T_\infty$), Φ becomes positive and increases with increase in optical thickness. On the other hand, if the plate surface is heated ($T_w > T_\infty$) Φ becomes negative and increases considerably in the negative direction.

Table 1:R_e =70559, P_m=153.1, P_r = 0.7, τ_o = 0.1, ε_o = 0.4, c=1, d=5, γ = 0.0186, b=21

θ_w	$G(\theta_w)$	$-\theta_0'(0)$	$-\theta_1'(0)$
¼	18.4725	0.1055	0.3306
½	15.8638	0.1055	0.2686
¾	13.4191	0.1055	0.2018
1	11.4363	0.1055	0.1424
2	7.3492	0.1055	0.0038
4	5.3032	0.1055	-0.0783

The first term on the right side of Eq. (38) represents convective heat transfer in the absence of radiation effects, while the second term denotes the first order radiation influence. It is observed that for the given set of parameters $-\theta_0'(0) = 0.1055$ and $\theta_2'(0) = 0.5076$ are found to remain the same. In this analysis, the variation of 'H' is studied with ϵ_w for the fixed values θ_w and ξ. the plate. The purpose of assuming θ_w constant is to minimize the coefficient $\frac{(\theta_w^4 - 1)}{(\theta_0 - 1)}$. The variation of H with ϵ_w is shown **in Fig. 8.** This figure exhibits that, when $\epsilon_w = 1$ the radiation interaction results in an increase in the convection heat transfer for $\theta_w < 2.1$, but it decreases for $\theta_w > 2.1$. This trend is similar to the observation made by Cess [20], but in his results the reversal from an increase to a decrease in convection heat transfer was found to occur for $\theta_w \approx 1.7$. The enhanced value of $\theta_w (\approx 2.1)$ is due to the presence of porous medium in this study.

Table 2 gives the comparative study of different physical quantities such as constant porosity, variable porosity, and variable conductivity of the medium. This table shows that the total heat flux in the variable porosity medium is about 79% more as compared to constant porosity medium. And the variable conductivity enhances the total heat flux by about 33% as compared to constant conductivity of the medium. It also shows that as ξ increases the total heat flux Ψ decreases.

4.6 Conclusions

1 The rise in temperature due to radiation transfer in a variable porosity medium is about 25% more as compared to constant porosity medium.

2. For higher values of b (=1650) the temperature profile decreases linearly.

3. The total heat flux in the variable porosity medium is about 79% more as compared to constant porosity medium.

4. The total heat flux in the presence of variable conductivity is about 33% more as compared to constant conductivity.

5. The total heat flux decreases with increase in ξ

	$G_{(w)}$	$-\theta_0'(0)$	$-\theta_1'(0)$	$\theta_2'(0)$	Ψ	ξ	Ψ
Constant Porosity	27.71	0.0497	4.069	6.146	3.080	0.01	4.210
Variable Porosity and Variable λ (d=10, γ=0.0217)	11.18	0.1433	0.268	0.5076	4.210	0.10	0.994
Absence of porous medium, matching with the results of Cess [20]			0.705	1.418	–	0.10	0.520
	4.70	0.332	–	–	–	0.15	0.384

Table 2: comparison of values for $R_e = 70559$, $p_m = 306.2$, $P_r = 0.7$, $\tau_o = 0.1$

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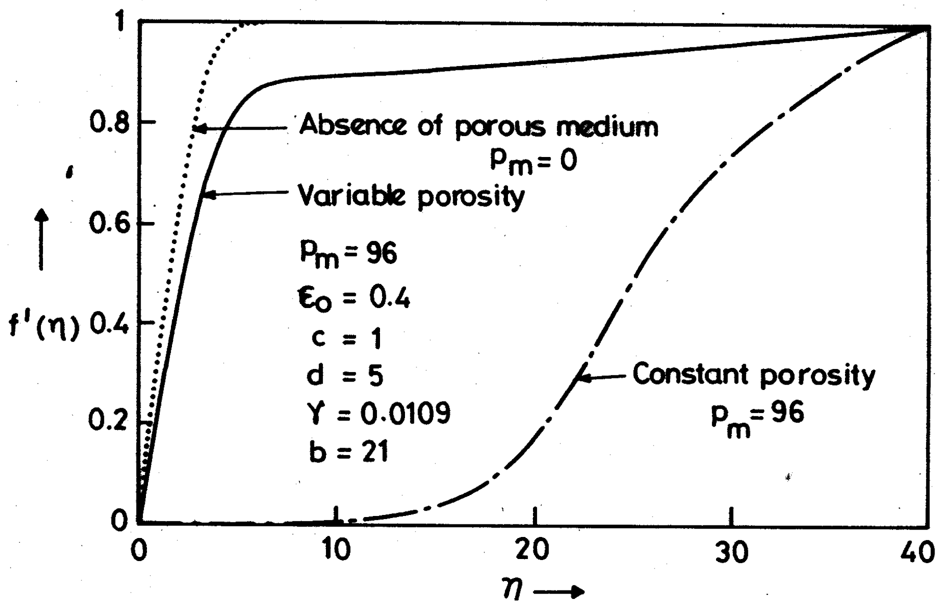

Fig. 2 Velocity Profile for Pr = 0.7, θ_w = 0.5, R_e = 7.0599 x 10^4.

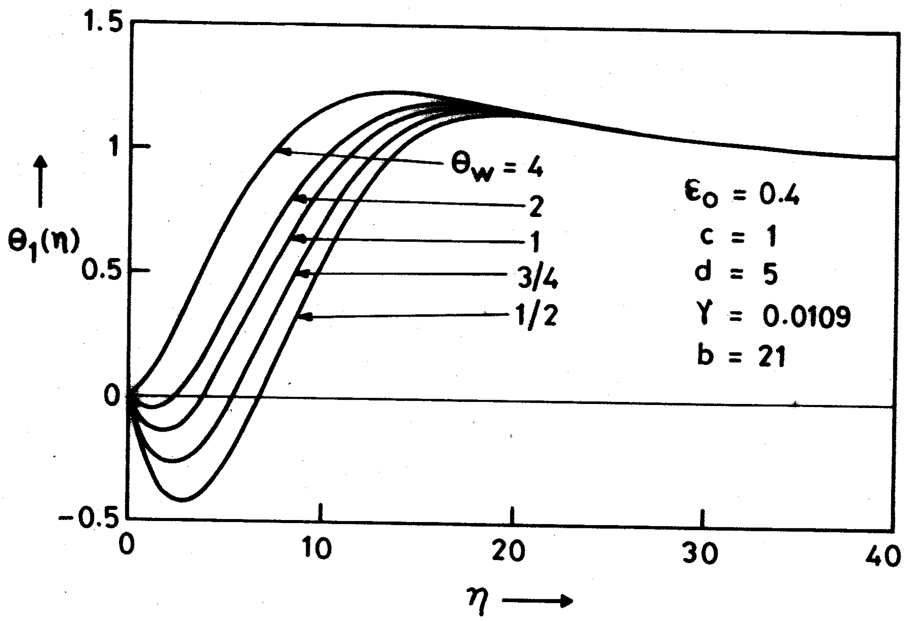

Fig. 3 Influence of θ_w on Temperature $\theta_1(\eta)$ for
P_m = 153, Pr = 0.7, R_e = 7.0599 x 10^4.

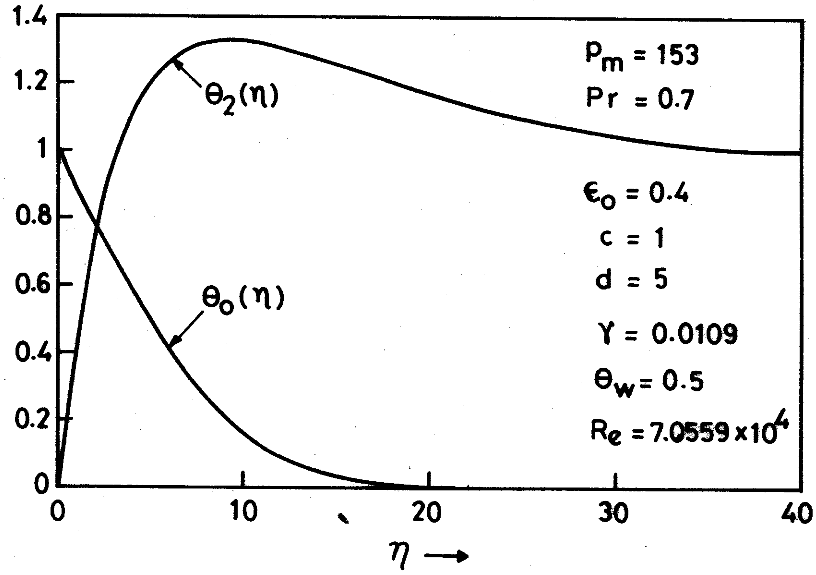

Fig. 4 Temperature Function in the presence of radiation $\theta_2(\eta)$ and absence of Radiation $\theta_0(\eta)$

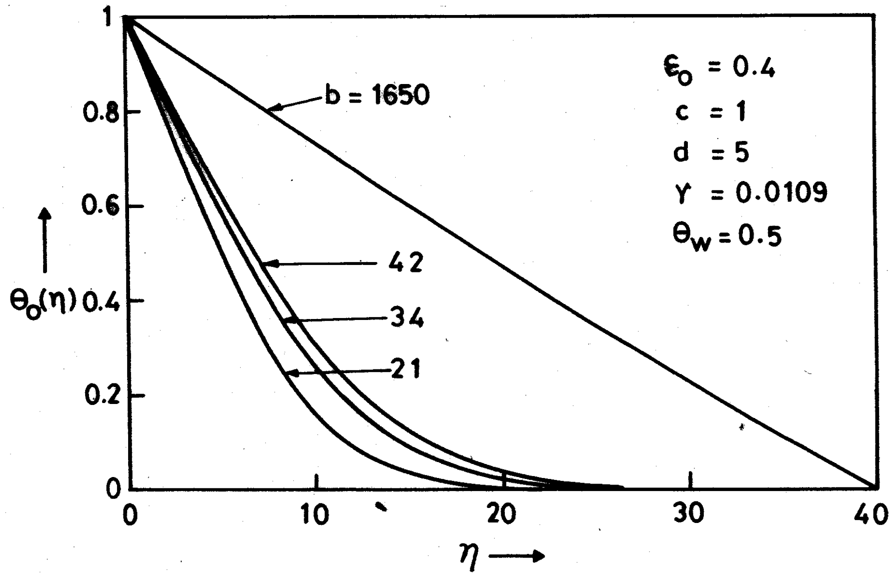

Fig. 5 Influence of 'b' on Temperature profile for $P_m = 153$, $Pr = 0.7$, $R_e = 7.0599 \times 10^4$.

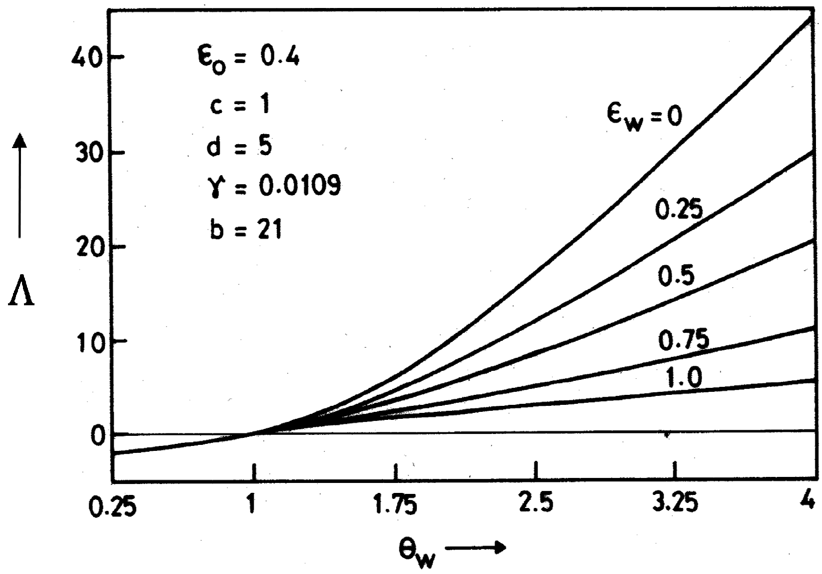

Fig. 6 Conductive Heat Transfer for $P_m = 153$, $Pr = 0.7$, $R_e = 7.0599 \times 10^4$.

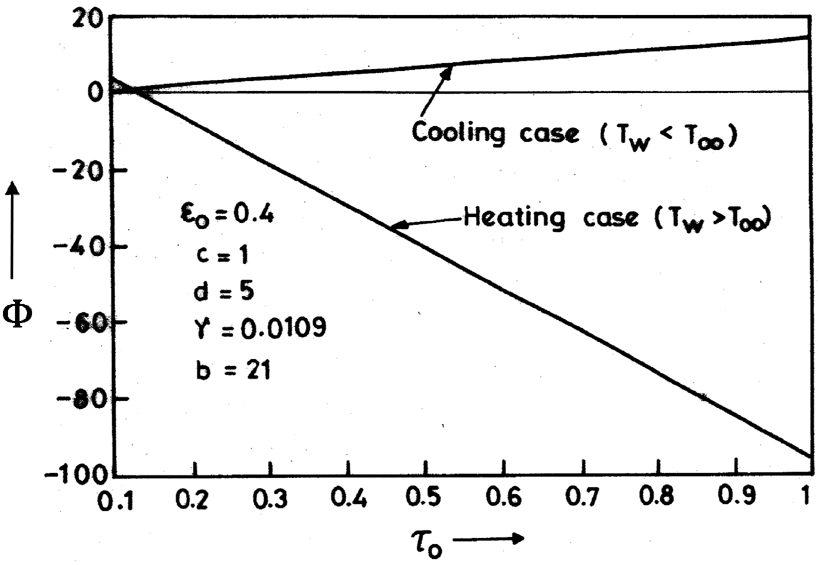

Fig. 7 Radiative Transfer for $P_m = 153$,

$Pr = 0.7$, $R_e = 7.0599 \times 10^4$.

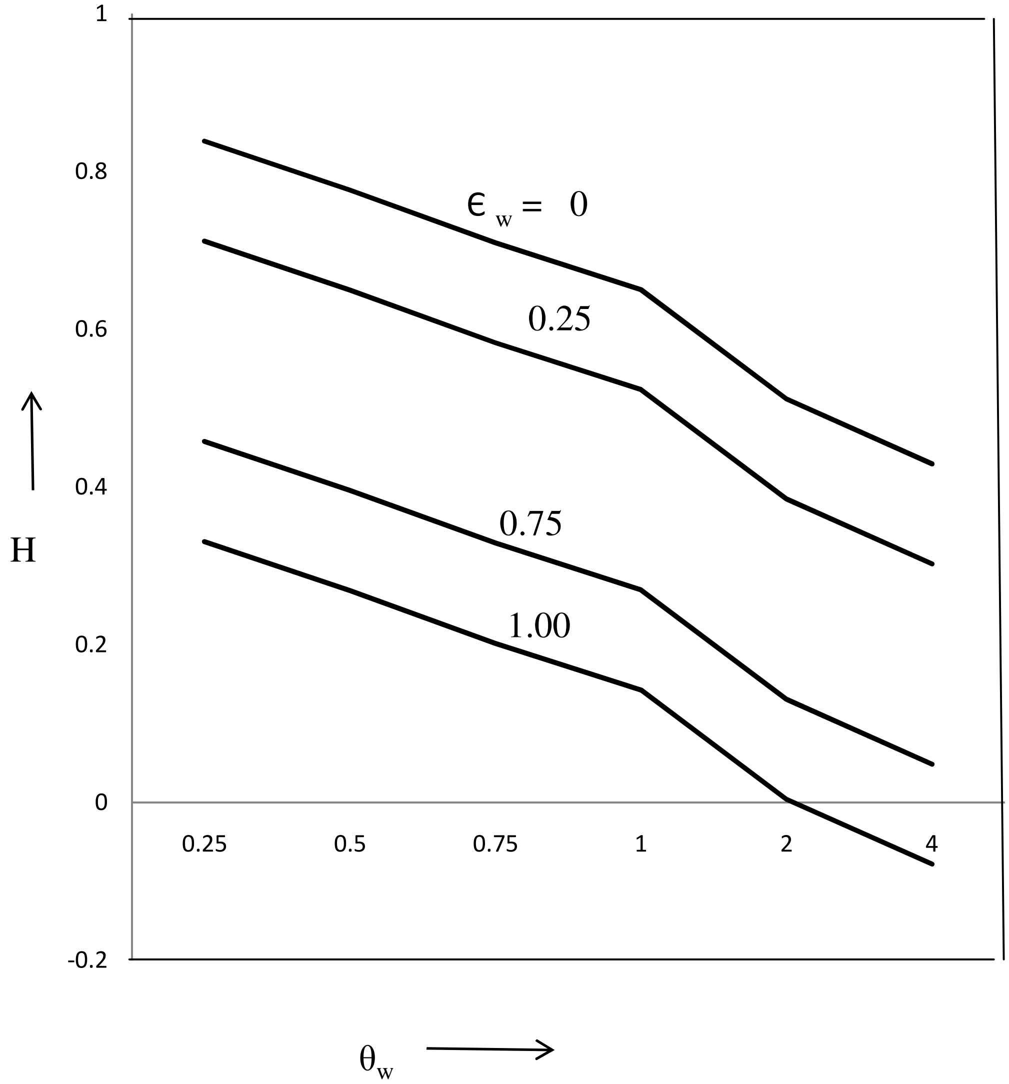

Fig. 8 Convective Heat Transfer for P_m=153, P_r=0.7, ε_0=0.4

C=1,d=5, γ=0.0109, b=21

CHAPTER 5

SUMMARY AND FUTURE WORK

5.1 Summary

This chapter highlights the important results of the problem analysed and the plan of future work. The severity of the energy and ecology problems facing man has incited him to look for alternate source and could be one of reasons for growing interest in buoyancy driven transport processes. Even though a number of articles are published on this topic, still many aspects remain unexplored. Therefore, the present study considers natural convection in a porous medium and determines the effects of different modes of heating from below the rectangular enclosure packed with spherical balls by keeping the upper boundary closed as well as open.

In this report one will find both the experimental and theoretical work. For the purpose of understanding this field of work, it has been divided into Five Chapters.

Chapter 1 deals with the need for the study, objective of the study, review of literature and the basic mechanism of Heat transfer, especially convection in general and flow and past through porous media in particular.

Chapter 2 describes the Basic equations, boundary conditions and a few well-known dimensionless parameters along with their special significance.

Chapter 3 highlights the Experiment conducted by the author and the results obtained. The actual apparatus used for the experiment is shown in the Figure. It consists of a rectangular wooden box of width 95 mm, depth 99 mm height 628 mm. Nevertheless, its cross sectional area is 85 mmX89 mm. The other important parts of the apparatus are heating and cooling walls at the bottom as well as at the top of the box. Surface temperature of the hot wall (Copper plate of 10 mm thickness) is maintained by using three strip heaters (Soldering iron) each having a maximum capacity of 25 W. And the surface temperature of the cold wall (Copper plate of 10 mm thickness) is kept at room temperature. 9 holes are made on one of the vertical walls of the box and each hole is fitted with a conical shaped cap to introduce the digital thermometer, which measures the temperature. Measurements are made for three solid particles, glass balls of diameter 15 mm and 10.85 mm and steel balls 9.5 mm respectively with air as the fluid. The porosity of the medium with these balls are respectively 0.415, 0.419 and 0.444. The Experimental results are discussed with figures and tables. The following important observations made in this experiment.

- Temperature distribution for iron balls and glass balls are similar, but the magnitude of temperature for iron balls is more, i.e., about 10% higher as compared to glass balls
- Temperature distribution in the absence of porous medium is more by about 7% as compared to glass balls
- The magnitude of the temperature in the vertical orientation is about 18% higher than that of the horizontal orientation

Chapter 4 gives the effect of radiation on boundary layer flow of an emitting, absorbing and scattering gas in a variable porosity medium. In this problem, a comparative study has been made for three different situations, namely (a) variable porosity (b) constant porosity and (c) absence of porous medium. In carrying out the solutions the momentum and energy equations are coupled and they are solved simultaneously by Runge-Kutta Gill method. The important results of this problem are as follows:

- The rise in temperature due to radiation transfer in a variable porosity medium is about 25% more as compared to constant porosity medium.
- For higher values of b (=1650) the temperature profile decreases linearly.
- The total heat flux in the variable porosity medium is about 79% more as compared to constant porosity medium.
- The total heat flux in the presence of variable conductivity is about 33% more as compared to constant conductivity.
- The total heat flux decreases with increase in ξ.

5.2 Future work

The future plan of work is to develop a theoretical model and to verify the same experimentally for flow through and past porous media. During the past forty years, the research of heat transfer in porous media has been the subject due to the increasing need for better understanding of the associated transport process in various engineering systems. Majority of the earlier studies on convective heat transfer in porous media were confined to either natural convection or forced convection and never both. Thus, the experimental study of mixed convection is still limited. Further, very few studies were reported on mixed convection dealing with theory followed by experimental verification or vice-versa. As far as the authors knowledge goes, the earlier study has not clarified the temperature measurement in a porous material. Technological applications in which porous materials are utilized include thermal energy, thermal insulation, heat transfer enhancement and heat exchangers. Such materials are favored because they produce high thermal performance and low pressure drop. To develop and optimize the design of such advanced devices, it is necessary to know the convective heat transfer coefficient data between a fluid and a solid material in a porous medium. However, there is a dearth of such data because of the technical difficulty of temperature measurements arising from the fact that the material structure is very fine and often complex. Thus, the present competitive society throws a number of challenges for scientific community. Therefore the challenge before the scientific community is to find a solution for the energy crisis. In this direction I am planning **to investigate non-linear aspects of the problem of fluid flow through and past porous media both theoretically and experimentally.**

APPENDIX

Conferences /Seminars/Publication/Reviewer

1. Presented a paper on “Effect of radiation on boundary layer flow of an absorbing, emitting and scattering gas in a variable porosity medium” at **International Conference on Frontiers of Fluid Mechanics** (ICFFM), Bangalore University, Bangalore, Aug 31- Sept 2, 2009
2. Presented a paper on “Fine structure constant- Determination” at the XXIV Annual National Convention of Indian Association of Physics Teachers’, at CSJM University Kanpur, Oct 10 -12, 2009
3. Presented a paper on “IUCAA SOLID STATE NIGHT SKY PHOTOMETER” at the **International Intra disciplinary Conference on the Frontiers of Astronomy**, Field Marshal KMCariappa College, Madikeri, Dec 28-30, 2009
4. Presented a paper on “Experimental study of natural convection in a Rectangular enclosure packed with porous media” at the 25th Annual Convention of Indian Association of Physics Teachers, Saurashtra Univesrsity, Rajkot,Oct 21–23, 2010
5. Presented a paper on “Experimental study of natural convection in porous Media heated from below” at Christ University, Bangalore, Feb 11 – 12, 2011

Publication

- Effect of radiation on Boundary layer flow of an Emitting, Absorbing and Scattering gas in a variable porosity medium. (Submitted for Heat and Mass Transfer: HMT-10-0427)

Reviewer – The following papers are reviewed for **Journal of Porous Media**,

Published by University of California, USA

1 Series solutions for the radiation-conduction interaction on unsteady by MHD flow, by I Ahmad, T Javed, T Hayat and M Sajid, 05-01-2009, Manuscript # 09- JPM-1575

2 Combined Radiation and Natural Convection within an open-ended porous channel-Validity of the Rosseland Approximation, by A Jabar, K Slimi and A Mhimid, 03-02-2010, Manuscript # 10-JPM-1782

3 Modeling of the heat transfer across porous honey comb structures, R Coquard,MThomas,BEstebe and D Bailis,31-01-2011,Manuscript# 11-JP2071