

V. V. K.

I Semester B.A./B.Sc. Examination, November/December 2017  
(CBCS) (F+R) (2014 -15 & Onwards)  
MATHEMATICS - I

Time : 3 Hours

Max. Marks : 70

*Instruction : Answer all questions.*

## PART - A

Answer any five questions.

(5×2 = 10)

1. a) Transform the matrix  $\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$  into  $\begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{bmatrix}$  using elementary transformations.

- b) Find the eigen values of the matrix  $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ . *-1, 4*

- c) Find the  $n^{\text{th}}$  derivative of  $\log_e (3x - 2)$ .

- d) If  $z = e^{\frac{x}{y}}$ , find  $\frac{\partial^2 z}{\partial x \partial y}$ .

- e) Evaluate  $\int_0^{\pi} \sin^5 x \, dx$ .  *$\frac{16}{15}$*

- f) Evaluate  $\int_0^{\infty} \frac{x^2}{(1+x^2)^2} \, dx$ .

- g) Show that the plane  $2x - 2y + z + 12 = 0$  touches the sphere  $x^2 + y^2 + z^2 - 2x - 4y - 2z - 3 = 0$ .

- h) Find 'k' so that the spheres  $x^2 + y^2 + z^2 + 4x + ky + 2z + 6 = 0$  and  $x^2 + y^2 + z^2 + 2x - 4y - 2z + 6 = 0$  may be orthogonal.



## PART – B

Answer **one** full question.

(1×15 = 15)

2. a) Find rank of the matrix by reducing in to echelon form.

$$\begin{bmatrix} 1 & 0 & 2 & -2 \\ 2 & -1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 4 & -1 & 3 & -1 \end{bmatrix}$$

- b) Solve completely the system of equations  $x + 3y - 2z = 0$ ;  $2x - y + 4z = 0$ ;  $x - 11y + 14z = 0$ .

- c) Find the eigen values and eigen vectors of the matrix  $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ .

OR

3. a) Find the values of  $\lambda$  and  $\mu$  such that the equations  $x + y + z = 6$ ;  $x + 2y + 3z = 10$ ;  $x + 2y + \lambda = \mu$  have i) a unique solution ii) infinite number of solutions.

- b) Reduce the matrix  $\begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \end{bmatrix}$  into normal form and find its rank.

- c) For the matrix  $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ , find  $A^{-1}$  using Cayley-Hamilton theorem.



PART – C

Answer two full questions.

(2×15 = 30)

4. a) Find the  $n^{\text{th}}$  derivative of  $\frac{x^2}{(x-1)^2(x-2)}$ .
- b) Find the  $n^{\text{th}}$  derivatives of  
i)  $(x^2 + 1)e^{5x}$       ii)  $\cos 3x \sin 4x$ .
- c) If  $y = (\sin^{-1}x)^2$ , prove that  $(1 - x^2) y_{n+2} - (2n + 1) xy_{n+1} - n^2 y_n = 0$ .

OR

5. a) If  $u = \log(x^3 + y^3 + z^3 - 3xyz)$ , show that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x + y + z}$ .
- b) State and prove Euler's theorem for homogeneous function of  $x$  and  $y$ .
- c) Find  $\frac{dz}{dt}$ , if  $z = \log(x^2 - y^2)$ , where  $x = e^t \cos t$ ,  $y = e^t \sin t$ .
6. a) If  $u = f(y - z, z - x, x - y)$ , show that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ .
- b) If  $u = \frac{yz}{x}$ ,  $v = \frac{xz}{y}$ ,  $w = \frac{xy}{z}$  show that  $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 4$ .
- c) For any positive integer  $n$ , prove that

$$\int_0^{\frac{\pi}{2}} \cos^n x dx = \frac{(n-1)(n-3)(n-5)\dots 2 \text{ or } 1}{n(n-2)(n-4)\dots 2 \text{ or } 1} \cdot R$$

Where  $R = \frac{\pi}{2}$ , if  $n$  is even

= 1, if  $n$  is odd.

OR



7. a) Obtain the reduction for  $\int \sec^n x dx$

b) Evaluate  $\int_0^{2a} x^2 \sqrt{2ax - x^2} dx$ .

c) Evaluate  $\int_0^{\infty} \frac{e^{-x} \sin \alpha}{x} dx$ , where  $\alpha$  is a parameter using Leibnitz's rule of differentiation under integral sign.

### PART - D

Answer **one full** question.

(1×15 = 15)

8. a) Find the equation of the plane passing through (2, 2, 1) and (9, 3, 6) and perpendicular to the plane  $2x + 6y + 6z = 9$ .

b) Show that the lines  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  and  $\frac{x-4}{5} = \frac{y-5}{6} = \frac{z-6}{7}$  are coplanar and find the point of intersection.

c) Find the equation of sphere passing through the points (3, 0, 0), (0, -1, 0), (0, 0, -2) and having its centre on the plane  $3x + 2y + 4z - 1 = 0$ .

OR

9. a) Show that the lines  $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{1}$  and  $\frac{x-2}{3} = \frac{y-2}{2} = \frac{z-6}{4}$  are coplanar. Find the equation of the plane containing these lines.

b) Derive the equation of a right circular cone in the standard form  $x^2 + y^2 = z^2 \tan^2 \alpha$ .

c) Find the equation of right circular cylinder whose radius is 4 units and passes through (1, -2, 3) and (3, -1, 1).

$$235 \times 10 = 2350$$

$$350 \times 9 = 3150$$

$$40 \times 7 = 280$$

$$\underline{\underline{5780}}$$

9100067877 (Parashuram)