

III Semester B.A./B.Sc. Examination, November/December 2017  
(Semester Scheme) (CBCS) (F+R) (2015-16 and Onwards)  
**MATHEMATICS – III**

Time : 3 Hours

Max. Marks : 70

**Instruction :** Answer all questions.**PART – A**

1. Answer any five questions :

**(5×2=10)**

a) Find the number of generators of the cyclic group of order 30.

b) Define right coset and left coset of a group.

c) Show that the sequence  $\left\{\frac{1}{n}\right\}$  is monotonically decreasing sequence.

d) State Raabe's Ratio test for convergence.

e) Test the convergence of the series :

$$1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \dots$$

f) Verify Rolle's theorem for the function  $f(x) = x^2 - 6x + 8$  in  $[2, 4]$ .

g) State Cauchy's mean value theorem.

h) Evaluate  $\lim_{x \rightarrow 0} \left( \frac{1 - \cos x}{x^2} \right)$ .**PART – B**

Answer one full question :

**(1×15=15)**2. a) If 'a and x' are any two elements of a group G then prove that  $O(a) = O(x a x^{-1})$ .b) Let G be a cyclic group of order d and 'a' be a generator, then prove that the element  $a^k$  ( $k < d$ ) is also a generator of G if and only if  $(k, d) = 1$ .

c) State and prove Fermat's theorem for groups.

OR

3. a) Prove that if 'a' is any element of a group G of order n then  $a^m = e$  for any integer m if and only if n divides m.
- b) Prove that every sub group of a cyclic group is cyclic
- c) Prove that every group of order less than or equal to 5 is abelian.

## PART - C

Answer two full questions :

(2×15=30)

4. a) Prove that the sequence  $\left\{ \frac{2n-7}{3n+2} \right\}$
- is monotonically increasing
  - is bounded.
- b) Prove that a monotonic increasing sequence bounded above is convergent.
- c) Show that the sequence  $\{x_n\}$  where  $x_1 = 1$  and  $x_n = \sqrt{2 + x_{n-1}}$  is convergent and converges to 2.

OR

5. a) Show that  $\{a_n\} = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!}$  is convergent.

b) Discuss the nature of the sequence  $\{x^{1/n}\}$ ,  $x > 0$ .

c) Examine the convergence of the sequences :

i)  $\frac{(n+1)^{n+1}}{n^n}$

ii)  $\left\{ \frac{2n^2 + 3n + 5}{n+3} \right\} \sin\left(\frac{\pi}{n}\right)$ .

6. a) State and prove D'Alembert's Ratio test for series of positive terms.

b) Test the convergence of the series  $1 + \frac{1}{2} + \frac{1.3}{2} + \frac{1.3.5}{2.4} + \dots$

c) Sum the series to infinity  $\frac{1}{5} - \frac{1.4}{5.10} + \frac{1.4.7}{5.10.15} - \frac{1.4.7.10}{5.10.15.20} + \dots$

OR

7. a) State and prove Cauchy's Root test for the convergence of series of positive terms.

b) Test the convergence of the series  $\sum \frac{1.2.3\dots n}{3.5.7.9\dots(2n+1)}$ .

c) Sum the series to infinity  $\frac{1}{6} + \frac{1.4}{6.12} + \frac{1.4.7}{6.12.18} + \dots$

PART - D

Answer **one full** question :

(1×15=15)

8. a) Prove that a function, which is continuous in a closed interval, takes every value between its bounds at least once.

b) Evaluate  $\lim_{x \rightarrow 0} \frac{e^{1/x}}{1 + e^{1/x}}$ .

c) Evaluate  $\lim_{x \rightarrow 0} (1 + \sin x)^{\cot x}$ .

OR

9. a) Examine the differentiability of the function  $f(x) = \begin{cases} x^2 - 1; & \text{for } x \geq 1 \\ 1 - x; & \text{for } x < 1 \end{cases}$   
at  $x = 1$ .

b) State and prove Lagrange's Mean value theorem.

c) Expand the function  $\log_e(1+x)$  up to the term containing  $x^4$  by Maclaurin's expansion.