NS – 305

I Semester B.A./B.Sc. Examination, November/December 2016
(CBCS) (F+R)
(2014-15 & Onwards)
MATHEMATICS – I

Time : 3 Hours   Max. Marks : 70

***Instruction** : Answer **all** questions.*

PART – A

Answer **any five** questions. **(5×2=10)**

1. a) Find the eigen values of the matrix $\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$.

   b) Find the value of λ for which the following system has a non trivial solution $2x - y + 2z = 0$, $3x + y - z = 0$ and $\lambda x - 2y + z = 0$.

   c) Find the n^{th} derivative of Sin^2x.

   d) If $z = x^3 - 4x^2y + 5y^2$ find $\frac{\partial^2 z}{\partial x dy}$.

   e) Evaluate $\int_0^{\pi/2} \cos^6 x dx$.

   f) Evaluate $\int_0^{\pi/2} \sin^7 x \cos^4 x \, dx$.

   g) Find the angle between the line $\frac{x-3}{2} = \frac{y-1}{1} = \frac{z+4}{-2}$ and the plane $x + y + z + 5 = 0$.

   h) Find the equation of the sphere having $(2, 1, -3)$ and $(1, -2, 4)$ as the ends of a diameter.

P.T.O.

PART – B

Answer **one** full question. **(1×15=15)**

2. a) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ 3 & 2 & 6 & 7 \end{bmatrix}$ by reducing to row reduced echelon form.

   b) Find the non trivial solution of the system $x + 3y - 2z = 0$, $2x - y + 4z = 0$ and $x - 11y + 14z = 0$.

   c) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

OR

3. a) Reduce the matrix $\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$ to the normal form and find its rank.

   b) Show that the system of equations $x + y + 2z = a$, $x + 3y - 2z = b$ and $5x + 7y + 6z = c$ is consistent, only when $c = 4a + b$ assuming this condition, express x, y in terms of a, b, z.

   c) Find the adjoint of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ using Cayley Hamilton theorem.

PART – C

Answer **two** full questions. **(2×15 = 30)**

4. a) Find the n^{th} derivative of $\frac{1}{6x^2 - 5x + 1}$.

   b) If $y = \tan^{-1}x$, show that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$.

   c) Find the n^{th} derivative of

      (i) $\log(x^2 - 4)$

      (ii) $\cos 2x \cos 3x$

OR

5. a) If $z = \sin(ax + y) + \cos(ax - y)$ prove that $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$.

b) State and prove Euler's theorem for homogeneous functions.

c) If $z = x^2 + y^2$ where $x = e^t \cos t$, $y = e^t \sin t$ find $\frac{dz}{dt}$.

6. a) If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

b) If $u = z - x$, $v = y - z$, $w = x + y + z$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.

c) Obtain the reduction formula for $\int \cos^n x\,dx$ where n is a positive integer.

OR

7. a) Obtain the reduction formula for $\int \sec^n x\,dx$ where n is a positive integer.

b) Evaluate $\int_0^\infty \frac{dx}{(1+x^2)^4}$.

c) Verify Leibnitz rule of differentiation under the integral sign for $\int_0^{\pi/2} \frac{dx}{\alpha(1+\cos x)}$ where α is a parameter.

PART – D

Answer **one** full question. **(1×15=15)**

8. a) Find the equation of the plane passing through the line of intersection of the planes $x + 2y + 3z - 4 = 0$, $2x + y - z + 5 = 0$ and perpendicular to the plane $5x + 3y + 6z + 8 = 0$.

b) Find K such that the lines $\frac{x-3}{1}=\frac{y-2}{3}=\frac{z-1}{4}$ and $\frac{x-4}{2}=\frac{y-2}{3}=\frac{z+2}{k}$ are Coplanar. For this K, find the plane containing the lines.

c) Find the equation of the sphere which passes through the points (0 0 0) (1 0 0) (0 1 0) and (0 0 1).

OR

9. a) Show that the Spheres $x^2 + y^2 + z^2 + 6y + 2z + 8 = 0$ and $x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$ Cut orthogonally and find their plane of intersection.

b) Find the shortest distance between the lines $\frac{x-2}{3}=\frac{y-6}{-2}=\frac{z-5}{-2}$ and $\frac{x-5}{2}=\frac{y-3}{-1}=\frac{z+4}{-6}$.

c) Find the equation of the right circular cylinder of radius 2 and whose axis is $\frac{x-1}{2}=\frac{y+3}{-1}=\frac{z-3}{5}$.