

NS – 307

III Semester B.A./B.Sc. Examination, November/December 2016
(Semester Scheme) (CBCS) (F + R)
(2015-16 and Onwards)
MATHEMATICS – III

Time : 3 Hours

Max. Marks : 70

Instruction : Answer all questions.

PART – A

Answer any five questions :

(5×2=10)

1. a) Write the order of the elements of the group of the cube roots of unity under multiplication.
- b) Find all the left cosets of the subgroup $H = \{0, 4, 8\}$ in (Z_{12}, \oplus_{12}) .
- c) Define a convergent sequence with an example.
- d) Show that $\left\{ \frac{3n+5}{2n+1} \right\}$ is monotonic decreasing sequence.
- e) Test the convergence of the series $\sqrt{\frac{1}{4}} + \sqrt{\frac{2}{6}} + \sqrt{\frac{3}{8}} + \dots$
- f) Prove that every differentiable function is a continuous function.
- g) State the Lagrange's mean value theorem.
- h) Evaluate : $\lim_{x \rightarrow a} \frac{x^a - a^x}{x^x - a^a}$.

PART – B

Answer one full question :

(1×15=15)

2. a) In a group G , prove that $O(a) = O(a^{-1})$, $\forall a \in G$.
- b) Prove that every subgroup of a cyclic group is cyclic.
- c) State and prove Fermat's theorem in groups.

OR

P.T.O.



- 3. a) If 'a' is a generator of a cyclic group of order 10, find the number of generators and write them.
- b) Prove that every group of order less than or equal to 5 is abelian.
- c) If 'n' is any positive integer and 'a' is relatively prime to n, then prove that $a^{\phi(n)} \equiv 1 \pmod{n}$.

PART - C

(2x15=30)

Answer two full questions :

- 4. a) If $\lim_{n \rightarrow \infty} a_n = a$ and $\lim_{n \rightarrow \infty} b_n = b$, prove that $\lim_{n \rightarrow \infty} (a_n \cdot b_n) = a \cdot b$.

- b) Discuss the nature of the sequence $\left\{ n^{\frac{1}{n}} \right\}$.

- c) Test the convergence of the sequences :

i) $\left\{ \sqrt{n+1} - \sqrt{n} \right\}$

ii) $\left\{ \frac{n + (-1)^n}{n} \right\}$.

OR

- 5. a) Prove that every convergent sequence is bounded.

- b) Discuss the convergence of the sequences :

i) $\left\{ \sqrt{n^2 + 1} - 1 \right\}$

ii) $\left\{ \frac{3 + 7 + 11 + \dots + (4n - 1)}{2n^2 + 3n} \right\}$.

- c) Find the limit of the sequence 0.5, 0.55, 0.555,

- 6. a) State and prove D'Alembert's Ratio Test for the series of positive terms.

b) Discuss the nature of the series : $\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots$

c) Sum to infinity of the series $1 + 2\left(\frac{1}{9}\right) + \frac{2 \cdot 5}{1 \cdot 2}\left(\frac{1}{81}\right) + \frac{2 \cdot 5 \cdot 8}{1 \cdot 2 \cdot 3}\left(\frac{1}{729}\right) + \dots$

OR

7. a) State and prove Cauchy's root test for a series of positive terms.

b) Discuss the convergence of the series $\frac{1}{1 \cdot 2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 4} + \frac{5}{3 \cdot 4 \cdot 5} + \dots$

c) Sum to infinity of the series :

$$1 + \frac{2^2}{1!} + \frac{3^2}{2!} + \frac{4^2}{3!} + \dots$$

PART - D

(1×15=15)

Answer **one full** question :

8. a) Prove that every continuous function over a closed interval is bounded.

b) Test the differentiability of $f(x) = \begin{cases} x^2, & \text{if } x \leq 3 \\ 6x - 9, & \text{if } x > 3. \end{cases}$ at $x = 3$.

c) Evaluate :

i) $\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right)$

ii) $\lim_{x \rightarrow \infty} \left(\frac{a^x + b^x + c^x + d^x}{4} \right)^{\frac{1}{x}}$.

OR

9. a) Prove that a function which is continuous in closed interval takes every value between its bounds at least once.

b) State and prove Rolle's theorem.

c) Expand $\log(1 + \sin x)$ upto the term containing x^4 by using Maclaurin's series.