I Semester B.A./B.Sc. Examination, November/December 2015
(2014-15 & Onwards)
(Semester Scheme) (CBCS) (F+R)
MATHEMATICS – I

Time : 3 Hours Max. Marks : 70

Instruction : Answer all questions.

PART – A

1. Answer any five questions : (5×2=10)

a) Find the value of K in order that the matrix

$$A = \begin{bmatrix} 6 & K & -1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$$ is of rank 2.

b) Find the value of λ for which the system of equations
$2x - y - 2z = 0$, $3x + y - z = 0$ and $\lambda x - 2y + z = 0$ has a non-trivial solution.

c) Find the n^{th} derivative of $e^{5x} + \sin 5x$.

d) If $z = x^3 - 3xy^2$ show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.

e) Evaluate $\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx$.

f) Evaluate $\int_0^{\pi/2} \cos^6 \theta \, d\theta$.

g) Find the angle between the planes $2x - y + 2z - 3 = 0$ and $3x + 6y + 2z - 4 = 0$

h) If two spheres $x^2 + y^2 + z^2 + 6z - k = 0$ and $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$ cut orthogonally. Find k.

PART – B

2. Answer any **one** full question : **(1×15=15)**

   a) Find the rank of the matrix

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 2 & 1 & 3 & 1 \\ \text{[illegible]} & 2 & 3 & 1 \end{bmatrix}$$ by reducing it to echelon form.

   b) Solve the system of equations $x + 2y + z = 3$, $2x + 3y + 3z = 10$ and $3x - y + 2z = 13$ by elimination method.

   c) State Cayley-Hamilton theorem and find the inverse of the matrix

$$\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$ by using it.

OR

3. a) Find the rank of the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$$ by reducing it to normal form.

   b) Find λ and μ such that the system of equations $x + 3y + 4z = 5$, $x + 2y + z = 3$ and $x + 3y + \lambda z = \mu$ has (i) no solution (ii) unique solution (iii) many solutions.

   c) Find the eigen values and the corresponding eigen vectors of the matrix

$$\begin{bmatrix} -3 & 8 \\ -2 & 7 \end{bmatrix}.$$

PART – C

4. Answer **any two full** questions : **(2×15=30)**

   a) Find the n^{th} derivative of $\dfrac{x}{(x-2)(x+3)}$.

b) Find the n^{th} derivative of

i) $y = x^2 e^{5x}$ ii) $y = \log(5x + 4)$

c) If $y = \tan^{-1}x$ prove that

$(1 + x^2)\, y_{n+2} + 2(n + 1)\, x y_{n+1} + n(n+1) y_n = 0$

OR

5. a) If $u = (x - y)^n + (y - z)^n + (z - x)^n$ prove that

$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

c) Find $\frac{df}{dt}$ where $f(x, y, z) = \log(x^2 + y^2 + z^2)$, $x = e^t$, $y = \sin t$, $z = \cos t$ by using partial differentiation.

6. a) If $z = f(x, y)$, $x = u - v$, $y = uv$, prove that $u\frac{\partial z}{\partial u} + v\frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} + 2y\frac{\partial z}{\partial y}$.

b) If $u = x + 3y^2 - z^3$, $v = 2x^2 - yz$, $w = 2z^2 - xy$. Evaluate $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at $(1, -1, 0)$.

c) Obtain the reduction formula for $\int \tan^n x\, dx$.

OR

7. a) Obtain the reduction formula for $\int \cos^n x\, dx$.

b) Evaluate : $\int_0^1 \frac{x^6}{\sqrt{1 - x^2}}\, dx$.

c) Evaluate : $\int_0^\infty \frac{e^{-x} \sin \alpha x}{x}$ where α is a Parameter using Leibnitz's rule of differentiation under the integral sign.

PART – D

8. Answer any one full question. (1×15=15)

a) Find the equation of the plane passing through the line of intersection of the planes $2x + y + 3z - 4 = 0$ and $4x - y + 2z - 7 = 0$ and perpendicular to the plane $x + 3y - 4z - 6 = 0$.

b) Prove that the lines $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{1}$ and $\frac{x-2}{3} = \frac{y-2}{2} = \frac{z-5}{4}$ are coplanar and find the equation of the plane containing them.

c) Find the equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0), (0, 0, 1) and whose centre lies on the plane $3x - y + z = 2$.

OR

9. a) Find the shortest distance between the skew lines

$$\frac{x}{2} = \frac{y}{-3} = \frac{z}{1} \text{ and } \frac{x-2}{3} = \frac{y-1}{-5} = \frac{z-2}{1}.$$

b) Derive the equation of a sign circular cone whose vertex is orgin, axis is z-axis and semi vertical angle is α. Hence obtain the equation of right circular cylinder whose vertex is at origin, axis is z – axis and semi vertical angle is 30°.

c) Find the equation of right circular cylinder whose radius is 4 units and was passes through (1, – 2, 3) and (3, – 1, 1).
