

V Semester B.A./B.Sc. Examination, November/December 2016
 (Semester Scheme)
 (Fresh) (CBCS) (2016 - 17 and Onwards)
 MATHEMATICS - VI

Time : 3 Hours

Max. Marks : 70

Instruction : Answer all questions.

PART - A

Answer any five questions.

(5×2 = 10)

1. a) Write Euler's equation when f is independent of y .

b) Show that the functional $I = \int_{x_1}^{x_2} (y^2 + x^2 y') dx$ assumes extreme values on the straight line $y = x$.

c) Define geodesic on a surface.

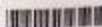
d) Evaluate $\int_C (5x dx + y dy)$ where C is the curve, $y = 2x^2$ from $(0, 0)$ to $(1, 2)$.

e) Evaluate $\int_0^{\pi} \int_0^{\sin y} y dx dy$.

f) Evaluate $\int_0^1 \int_0^x \int_0^z dy dz dx$,

g) Show that the area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab using Green's theorem.

h) Evaluate using Stoke's theorem $\oint_C (yz dx + zx dy + xy dz)$ where C is the curve $x^2 + y^2 = 1, z = y^2$.



PART - B

Answer two full questions :

(2x10=20)

2. a) Prove that the necessary condition for the integral $I = \int_{x_1}^{x_2} f(x, y, y') dx$ with

$$y(x_1) = y_1 \text{ and } y(x_2) = y_2 \text{ to be an extremum is } \frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0.$$

b) Find the geodesic on a plane.

OR

3. a) Show that the extremal of $I = \int_{x_1}^{x_2} \sqrt{y(1+(y')^2)} dx$ is a parabola.

b) Find the extremal of the functional $I = \int_0^1 \sqrt{1+(y')^2} dx$ with $y(0) = 1$ and $y(1) = 2$.

4. a) Find the shape of a chain which hangs under gravity between two fixed points.

b) Find the extremal of the functional $\int_0^1 [(y')^2 + x^2] dx$ subject to constraint

$$\int_0^1 y dx = 2 \text{ and having end conditions } y(0) = 0, y(1) = 1.$$

OR

5. a) Find the function y which makes the integral $I = \int_{x_1}^{x_2} [y^2 + 4(y')^2] dx$ an extremum

b) Find the extremal of the functional $I = \int_0^\pi [(y')^2 - y^2] dx$ with $y(0) = 0$ and

$$y(\pi) = 1 \text{ and subject to the constraint } \int_0^\pi y dx = 1.$$

PART - C

Answer two full questions :

(2x10=20)

6. a) Evaluate $\int_0^1 \int_0^1 (x+y+z) \, ds$ where C is the line joining the points (0, 1, 0) and (1, 2, 3).

b) Evaluate $\iint_A (4x^2 - y^2) \, dx \, dy$, where A is the area bounded by the lines $y = 0$, $y = x$ and $x = 1$.

OR

7. a) Evaluate $\int_0^{\pi} \int_0^{\pi} x e^{x/y} \, dx \, dy$, by changing the order of integration.

b) Find the area bounded by the arc of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in first quadrant.

8. a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz \, dy \, dx}{\sqrt{1-x^2-y^2-z^2}}$.

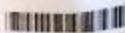
b) Evaluate $\iint_R \frac{x^2 y^2}{x^2 + y^2} \, dx \, dy$ using polar co-ordinates, where R is the annular region between the circles $x^2 + y^2 = 2$ and $x^2 + y^2 = 1$.

OR

a) Find the volume bounded by the surface $z = a^2 - x^2$ and the planes $x = 0$, $y = 0$, $z = 0$ and $y = b$.

b) If R is the region bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$,

show that $\iiint_R z \, dx \, dy \, dz = \frac{1}{24}$.



PART - D

Answer two full questions :

(2x10=20)

10. a) State and prove Gauss' Divergence Theorem.

b) Evaluate using Green's theorem for $\oint_C [xy \, dx + yx^2 \, dy]$, where C is the curve enclosing the region bounded by the curve $y = x^2$ and the line $y = x$.

OR

11. a) Verify Green's theorem in the plane for $\oint_C [(x^2 - xy^2) \, dx + (y^2 - 2xy) \, dy]$, where C is the square with vertices (0, 0), (2, 0), (2, 2) and (0, 2).

b) Evaluate $\iiint_S \vec{F} \cdot \hat{n} \, ds$ using divergence theorem where $\vec{F} = x\hat{i} - y\hat{j} + (z^2 - 1)\hat{k}$ and S is the closed surface bounded by planes $z = 0$, $z = 1$ and the cylinder $x^2 + y^2 = 4$.

12. a) Verify Stokes theorem for $\vec{F} = 2y\hat{i} + 3x\hat{j} - z^2\hat{k}$; C is the boundary of the upper half of the surface of the sphere $x^2 + y^2 + z^2 = 9$.

b) Evaluate using Gauss' divergence theorem $\iiint_S \vec{F} \cdot \hat{n} \, ds$, where

$\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and S is the total surface of the rectangular parallelepiped bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x = 1$, $y = 2$, $z = 3$.

OR

13. a) Evaluate $\oint_C \vec{F} \cdot d\vec{r}$, using Stoke's theorem where $\vec{F} = (y - z + 2)\hat{i} + (yz)\hat{j} - (xz)\hat{k}$ taken over the surface S of the cube $0 \leq x \leq 2$, $0 \leq y \leq 2$, $0 \leq z \leq 2$.

b) By using Green's theorem evaluate $\oint_C [(3x - y) \, dx + (2x + y) \, dy]$ where C the circle $x^2 + y^2 = a^2$.