UN – 174

III Semester B.A./B.Sc. Examination, November/December 2015
(Semester Scheme) (CBCS) (Fresh)
(2015-16 & Onwards)
MATHEMATICS – III

Time : 3 Hours   Max. Marks : 70

***Instruction** : Answer **all** questions.*

PART – A

1. Answer **any five** questions. **(5×2=10)**
   a) Find the number of generators of the cyclic group of order 24.
   b) Find all the left cosets of the subgroup H = {0, 2, 4} of the group $(Z, \oplus_6)$.
   c) Show that $\left\{\frac{1}{n}\right\}$ is monotonically decreasing sequence.
   d) Test the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^n$.
   e) Discuss the convergence of the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$.
   f) State Cauchy's mean value theorem.
   g) Verify Rolle's theorem for the function $f(x) = 8x - x^2$ in [2, 6].
   h) Evaluate $\lim_{n\to 0}(\text{cosec}x - \cot x)$.

PART – B

Answer **any one full** question. **(1×15=15)**

2. a) If 'a' and 'b' are any 2 arbitrary elements of a group G, then prove that $O(a) = O(bab^{-1})$.
   b) Show that the group {1, 2, 3, 4, 5, 6} under $\otimes_7$ is cyclic and find the number of generators.
   c) State and prove Lagranges theorem in groups.

OR

3. a) Prove that any 2 right (left) cosets of subgroup H of a group G are either identical or disjoint.

   b) Define cyclic group. Show that every cyclic group is abelian.

   c) If an element 'a' of a group G is of order n and e is the identity in G, then prove that for some positive integer m, $a^m = e$ if and only if n divides m.

PART – C

Answer **two full** questions. **(2×15=30)**

4. a) If $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, then prove that $\lim_{n\to\infty} a_n b_n = ab$.

   b) Discuss the nature of the sequence $\left\{x^{1/n}\right\}$, $x > 0$.

   c) Examine the convergence of the sequences

      i) $\left\{\dfrac{1+(-1)^n n}{n+1}\right\}$

      ii) $\left\{\sqrt{n}\left(\sqrt{n+4}-\sqrt{n}\right)\right\}$.

OR

5. a) Prove that a monotonic decreasing sequence which is bounded below is convergent.

   b) Show that the sequence $\{x_n\}$ defined by $x_n = \dfrac{1}{n+1}+\dfrac{1}{n+2}+\ldots+\dfrac{1}{n+n}$ is convergent.

   c) Examine the behaviour of the sequences

      i) $\left\{\left(\dfrac{n+1}{n}\right)^n (n+1)\right\}$

      ii) $n\,[\log(n+1) - \log n]$.

6. a) State and prove D'Alembert's Ratio test for series of positive terms.

   b) Test the convergence of the series $\sum \frac{1.5.9...(4n-3)}{3.7.11...(4n-1)}$.

   c) Sum the series to infinity $1+\frac{1}{15}+\frac{1.6}{15.30}+\frac{1.6.11}{15.30.45}+\ldots$.

OR

7. a) State and prove Cauchy's root for the convergence of series of positive terms.

   b) Discuss the convergence of the series $\sum\left(\sqrt{n^4+1}-\sqrt{n^4-1}\right)$.

   c) Sum the series to infinity $1+\frac{2^2}{1!}+\frac{3^2}{2!}+\frac{4^2}{3!}$.

PART – D

Answer **any one full** question. **(1×15=15)**

8. a) Discuss the continuity of $f(x)=\begin{cases}\dfrac{e^{1/x^2}}{1-e^{1/x^2}} & \text{for } x\neq 0\\ 1 & \text{for } x=0\end{cases}$ at $x = 0$.

   b) State and prove Lagranges mean value theorem.

   c) Evaluate $\lim\limits_{x\to\pi/2}(\sin x)^{\tan x}$.

OR

9. a) Prove that a function which is continuous on a closed interval is bounded.

   b) Examine the differentiability of the function

   $f(x)=\begin{cases}x^2-1 & \text{for } x\geq 1\\ 1-x & \text{for } x<1\end{cases}$ at $x = 1$.

   c) Obtain Maclaurin's expansion of log (1 + sin x) upto the term containing x^4.