

I Semester B.A./B.Sc. Examination, Nov./Dec. 2014
(2014-15 and Onwards) (Semester Scheme) (CBCS) (Fresh)
MATHEMATICS - I

SN - 249

Time : 3 Hours

Max. Marks : 70

Instruction: Answer all questions.

PART - A

1. Answer any five questions :

(5×2=10)

a) Define :

i) Equivalent matrices

ii) Row reduced Echelon form of a matrix.

b) Find the value of λ , for which the system $3x + y - \lambda z = 0$; $4x - 2y - 3z = 0$; $2\lambda x + 4y + \lambda z = 0$ has a non-trivial solution.

c) Find the n^{th} derivative of $\sin^2 x$.

d) If $z = \cos^{-1}(xy)$ prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$.

e) Using reduction formula, evaluate $\int \sin^3 x dx$.

f) Evaluate $\int_0^{\pi} x \cos^4 x dx$.

g) Show that the planes $x + 2y - 3z + 4 = 0$ and $4x + 7y + 6z + 2 = 0$ are perpendicular.

h) Write the condition for co-planarity of two lines.



PART - B

(1×15=15)

Answer **any one full** question.

2. a) Find the rank of the matrix

$$\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

b) Solve completely the system equations $x + 3y - 2z = 0$; $2x - y + 4z = 0$;
 $x - 11y + 14z = 0$.

c) Verify Cayley-Hamilton's theorem and hence find inverse of the matrix

$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

OR

3. a) Reduce the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$ into normal form and hence find the rank.b) Show that the following system of equations is consistent and solve them
 $x + 2y + 2z = 1$; $2x + y + z = 2$; $3x + 2y + 2z = 3$; $y + z = 0$.

c) Find the eigen values and the corresponding eigen vectors of the matrix

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix}$$

PART - C

Answer **any two full** questions.

(2×15=30)

4. a) Find the n^{th} derivative of $\frac{4x}{(x+1)^2(x-1)}$.

b) State and prove Leibnitz's theorem.

c) If $y = \sin^{-1}x$ show that $(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - n^2 y_n = 0$.

OR

5. a) If $z = \sin(ax + y) + \cos(ax - y)$ prove that $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$.
- b) State and prove Euler's theorem for homogeneous functions.
- c) Find the total derivative of u w.r.t 't' where $u = e^x \sin y$ where $x = \log t$, $y = t^2$.
6. a) If $u = f(r)$ where $r^2 = x^2 + y^2 + z^2$ prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = f''(r) + \frac{2}{r} f'(r)$.
- b) If $u = z - x$, $v = y - z$ and $w = x + y + z$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.
- c) Obtain the reduction formula for $\int \sin^n x dx$ where n is a positive integer.

OR

7. a) Obtain the reduction formula for $\int \tan^n x dx$.

b) Evaluate :

i) $\int_0^{\frac{2}{5}} x^2 \sqrt{2-x} dx$

ii) $\int_0^{\pi} x \sin^5 x \cdot \cos^4 x dx$.

c) Evaluate $\int_0^1 \frac{x^a - 1}{\log x} dx$.

Where a is a parameter.

PART - D

Answer **any one full** question.

(1×15=15)

8. a) Find the equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 4$, $2x + y - z + 5 = 0$ and perpendicular to the plane $5x + 3y + 6z + 8 = 0$.
- b) Obtain the condition for co-planarity of two lines in both vector and Cartesian forms.



- c) Find the equation of the right circular cone through the point $(2, 1, 3)$ with vertex at the point $(1, 1, 2)$ and axis parallel to the line

$$\frac{x-2}{2} = \frac{y-1}{-4} = \frac{z+2}{3}$$

OR

9. a) Find the shortest distance between the skew lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and

$$\frac{x-2}{3} = \frac{y-6}{4} = \frac{z-5}{5}$$

- b) Show that the plane $2x + 3y + 4z = 58$ touches the sphere $x^2 + y^2 + z^2 - 4x - 6y - 8z = 0$ and find the point of contact.
- c) Find the equation of the right circular cylinder whose axis is the line

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n} \text{ and whose radius is 'r'.$$
