THEORY OF BALLISTIC GALVANOMETER

B.G. consists of a rectangular coil of area A and number of turns N suspended in a uniform magnetic field B. Let it carry a current I for a short interval of time dt. Then the momentary torque on the coil is given by

$$\vec{T} = NI\left(\vec{A} \times \vec{B}\right)$$

As the plane of the coil is always parallel to B,

$$T = N I A B$$

Angular impulse given to the coil = change in angulr momentum of the coil.

i.e., $N A B \int_0^T I\, dt = I'\omega$, where I' is the M.I of the coil about its axis of suspension and w is the angular velocity of the coil.

$$\Rightarrow \quad N A B q = I^1 \omega \Rightarrow \omega = \frac{N A B q}{I} \text{ ----- (1)}$$

K.E of the coil = P.E of the suspension wire (As the coil rotates, the suspension wire get twisted and hence K.E of the coil is converted into P.E of the wire)

i.e., $\frac{1}{2} I' \omega^2 = \frac{1}{2} C \theta_0^2$, where C is the couple per unit twist of the suspension wire and θ_0 is the first throw of the light spot on the scale.

$$\Rightarrow \quad \omega^2 = \frac{C\theta_0^2}{I'} \Rightarrow \frac{N^2 A^2 B^2 q^2}{I'^2} = \frac{C\theta^2}{I'}$$

$$\Rightarrow \quad q^2 \equiv \frac{I' C \theta_0^2}{N^2 A^2 B^2} \Rightarrow q^2 = \frac{C^2}{N^2 A^2 B^2} \frac{I'}{C} \theta_0^2 \text{ ----- (2)}$$

The period of oscillation of the coil is given by

$$T = 2\pi \sqrt{\frac{I'}{C}} \Rightarrow T^2 = 4\pi^2 \frac{I'}{C}$$

$$\Rightarrow \frac{I'}{C} = \frac{T^2}{4\pi^2} \text{ ------ (3)}$$

Substituting (3) in (2), we get :

$$q^2 = \frac{C^2}{N^2 A^2 B^2} \frac{T^2}{4\pi^2} \theta_0^2 \Rightarrow \boxed{q = \frac{C}{NAB} \frac{T}{2\pi} \theta_0} \text{ ------- (4)}$$

or $\boxed{q = K \frac{T}{2\pi} \theta_0}$ or $\boxed{q = K' \theta_0}$

where $K = \frac{C}{NAB}$ is a constant for the B.G and K' is also a constant called the ballistic constant or ballistic reduction factor of B.G.

Charge sensitivity of a B.G : It is defined as the deflection produced in the coil per unit change.

If a change q passing through B.G. produces a deflection θ_0, then charge sensitivity = $\frac{\theta_0}{q}$.

We have $q = \frac{C}{NAB}\frac{T}{2\pi}\theta_0$

$$\therefore \boxed{\frac{\theta_0}{q} = \frac{NAB}{C}\frac{2\pi}{T}} \text{ ------- (5)}$$

Current sensitivity of a B.G :

It is defined as the deflection produced in the coil per unit current. If a current I through the coil produces a deflection θ_0, then current sensitivity = $\frac{\theta_0}{I}$.

We have $T = C\theta_0$

$\Rightarrow \quad N I A B = C\theta_0$

$$\Rightarrow \boxed{\frac{\theta_0}{I} = \frac{NAB}{C}} \text{ ------- (6)}$$

From (5) and (6), $\frac{\theta_0}{q} = \frac{\theta_0}{I} \times \frac{2\pi}{T}$ ------- (7)

$$\boxed{\therefore \quad \text{Charge sensitivity} = \frac{2\pi}{T} \times \text{Current sensitivity}}$$

Damping :

When the coil of B.G rotates on passing a current through it, the entire K.E of the coil is not used to twist the suspension wire as a part of the K.E is used in overcoming damping force like air resistance, electromagnetic damping etc., ∴ the deflection of the coil successively decreases. If $\theta_1, \theta_3, \theta_5$, ... are the deflections of the light spot to the left and $\theta_2, \theta_4, \theta_6$, ... are the deflections to the right, then it is observed that

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = d, \text{ a constant.}$$

'd' is called the decrement and logarithm of it is called logarithmic decrement, λ

$$\lambda = \log_e d \qquad \text{or} \qquad d = e^{\lambda}.$$

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = e^{\lambda}.$$

For one complete oscillation (i.e., for two successive throws on the same side),

$$\frac{\theta_1}{\theta_3} = \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} = d^2 = e^{2\lambda}.$$

i.e., log . decrement for one oscillation = $e^{2\lambda}$.

\ log . decrement for $\frac{1}{4}$th of an oscillation = $e^{\lambda/2}$.

If θ_0 is the first correct throw (without damping) on one side and θ is the first observed throw (with damping) on the same side, then

$$\frac{\theta_0}{\theta} = e^{\lambda/2} \quad \text{or} \quad \theta_0 = \theta\, e^{\lambda/2} \Rightarrow \theta_0 = \theta\left(1 + \frac{\lambda}{2}\right) \text{ ------ (8)}$$

$\because \lambda$ is small, higher order terms are neglected

Substituting (8) in (4), we get

$$\boxed{q = \frac{C}{NAB}\ \frac{T}{2\pi}\ \theta \left(1+\frac{\lambda}{2}\right)}$$

Thus the correction for the deflection due to damping is obtained.

Calculating l

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \cdots\cdots = \frac{\theta_n}{\theta_{n+1}} = e^{\lambda}$$

$$\frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} \times \cdots\cdots \times \frac{\theta_n}{\theta_{n+1}} = e^{n\lambda}$$

i.e., $\frac{\theta_1}{\theta_{n+1}} = e^{n\lambda} \Rightarrow n\lambda = \log_e \frac{\theta_1}{\theta_{n+1}}$

$$\Rightarrow n\lambda = 2.303 \log \frac{\theta_1}{\theta_{n+1}}$$

$\Rightarrow \boxed{\lambda = \frac{2.303}{n} \log \frac{\theta_1}{\theta_{n+1}}}$, where n gives the no. of successive throws on one side.

Ex. : For two successive throws on one side, $\frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} = e^{2\lambda} = \frac{\theta_1}{\theta_3}$

$$2\lambda = \log_\varepsilon \frac{\theta_1}{\theta_3} = 2.303 \log \frac{\theta_1}{\theta_3}$$

$$\boxed{\lambda = \frac{1}{2} \times 2.303 \log \frac{\theta_1}{\theta_3}}$$

Applications : (1) High resistance by leakage

The ckt connections are made as shown in the figure. With K open, K, is close for a known interval of time so that C is charged to a value of q_0. Then the capacitor is discharged immediately through the B.G by closing K_2 and the corresponding throw θ_0 is noted.

Then $q_0 = CV_0 = K^1 \theta_0 \left(1+\frac{l}{2}\right)$ ------ (1), where V_0 is the p.d across the capacitor.

The capacitor is again charged to the same value q_0. Then it is allowed to discharg through the high resistance R for a known interval of time by pressive the key K. The remaining charge q on the capacitor is discharged through the B.G by closing K_2 and the corresponding through θ is noted. Then

$q = CV = K^1 \theta \left(1+\frac{\lambda}{2}\right)$, ------ (2), where V is the p.d across the capacitor.

$$\frac{q_0}{q} = \frac{V_0}{V} = \frac{\theta_0}{\theta}$$

We know that $q = q_0 e^{-\frac{t}{RC}}$ for the discharge of a capacitor d is the time for which capacitor is discharged through R.

$$\Rightarrow \frac{q_0}{q} = e^{-\frac{t}{RC}} \Rightarrow \frac{q_0}{q} = e^{\frac{t}{RC}} \Rightarrow \frac{\theta_0}{\theta} = e^{\frac{t}{RC}}$$

$$\Rightarrow \frac{t}{RC} = \log_e \frac{\theta_0}{\theta}$$

$$\Rightarrow R = \frac{t}{C \log \log_e \frac{\theta_0}{\theta}} \Rightarrow \boxed{R = e \frac{t}{2.303\, C \log \frac{\theta_0}{\theta}}}$$

A graph o $\log \frac{\theta_0}{\theta}$ vs t is plotted slope is calculated.

Then $R = \frac{\text{Slope}}{2.303\ C}$