

SOME IMPORTANT MATHEMATICAL FORMULAE

Circle : Area = πr^2 ; Circumference = $2\pi r$.

Square : Area = x^2 ; Perimeter = $4x$.

Rectangle: Area = xy ; Perimeter = $2(x+y)$.

Triangle : Area = $\frac{1}{2}$ (base)(height); Perimeter = $a+b+c$.

$$\text{Area of equilateral triangle} = \frac{\sqrt{3}}{4} a^2.$$

Sphere : Surface Area = $4\pi r^2$; Volume = $\frac{4}{3}\pi r^3$.

Cube : Surface Area = $6a^2$; Volume = a^3 .

Cone : Curved Surface Area = πrl ; Volume = $\frac{1}{3}\pi r^2 h$

$$\text{Total surface area} = \pi r l + \pi r^2$$

Cuboid : Total surface area = $2(ab + bh + lh)$; Volume = lbh .

Cylinder : Curved surface area = $2\pi rh$; Volume = $\pi r^2 h$

$$\text{Total surface area (open)} = 2\pi rh;$$

$$\text{Total surface area (closed)} = 2\pi rh + 2\pi r^2.$$

SOME BASIC ALGEBRAIC FORMULAE:

$$1. (a + b)^2 = a^2 + 2ab + b^2.$$

$$2. (a - b)^2 = a^2 - 2ab + b^2.$$

$$3. (a + b)^3 = a^3 + b^3 + 3ab(a + b).$$

$$4. (a - b)^3 = a^3 - b^3 - 3ab(a - b).$$

$$5. (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca.$$

$$6. (a + b + c)^3 = a^3 + b^3 + c^3 + 3a^2b + 3a^2c + 3b^2c + 3b^2a + 3c^2a + 3c^2b + 6abc.$$

$$7. a^2 - b^2 = (a + b)(a - b).$$

$$8. a^3 - b^3 = (a - b)(a^2 + ab + b^2).$$

$$9. a^3 + b^3 = (a + b)(a^2 - ab + b^2).$$

$$10. (a + b)^2 + (a - b)^2 = 4ab.$$

$$11. (a + b)^2 - (a - b)^2 = 2(a^2 + b^2).$$

$$12. \text{If } a + b + c = 0, \text{ then } a^3 + b^3 + c^3 = 3abc.$$

INDICES AND SURDS

$$1. a^m a^n = a^{m+n} \quad 2. \frac{a^m}{a^n} = a^{m-n} \quad 3. (a^m)^n = a^{mn} \quad 4. (ab)^m = a^m b^m$$

$$5. \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \quad 6. a^0 = 1, \quad a \neq 0. \quad 7. a^{-m} = \frac{1}{a^m} \quad 8. a^x = a^y \Rightarrow x = y$$

$$9. a^x = b^x \Rightarrow a = b \quad 10. \sqrt{a \pm 2\sqrt{b}} = \sqrt{x} \pm \sqrt{y}, \text{ where } x + y = a \text{ and } xy = b.$$

LOGARITHMS

$$a^x = m \Rightarrow \log_a m = x \quad (a > 0 \text{ and } a \neq 1)$$

1. $\log_a mn = \log m + \log n.$
2. $\log_a \left(\frac{m}{n} \right) = \log m - \log n.$
3. $\log_a m^n = n \log m.$
4. $\log_b a = \frac{\log a}{\log b}.$
5. $\log_a a = 1.$
6. $\log_a 1 = 0.$
7. $\log_b a = \frac{1}{\log_a b}.$
8. $\log_a 1 = 0.$
9. $\log(m+n) \neq \log m + \log n.$
10. $e^{\log x} = x.$
11. $\log_a a^x = x.$

PROGRESSIONS**ARITHMETIC PROGRESSION**

$a, a + d, a + 2d, \dots$ are in A.P.

n^{th} term, $T_n = a + (n-1)d.$

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d].$

If a, b, c are in A.P, then $2b = a + c.$

GEOMETRIC PROGRESSION

a, ar, ar^2, \dots are in G.P.

Sum to n terms, $S_n = \frac{a(1-r^n)}{1-r}$ if $r < 1$ and $S_n = \frac{a(r^n-1)}{r-1}$ if $r > 1.$

Sum to infinite terms of G.P, $S_\infty = \frac{a}{1-r}.$

If a, b, c are in A.P, then $b^2 = ac.$

HARMONIC PROGRESSION

Reciprocals of the terms of A.P are in H.P

$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \dots$ are in H.P

If a, b, c are in H.P, then $b = \frac{2ac}{a+c}.$

MATHEMATICAL INDUCTION

$$1 + 2 + 3 + \dots + n = \sum n = \frac{n(n+1)}{2}.$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \sum n^2 = \frac{n(n+1)(2n+1)}{6}.$$

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$$1^3 + 2^3 + 3^3 + \dots + n^3 = \sum n^3 = \frac{n^2(n+1)^2}{4}.$$

PERMUTATIONS AND COMBINATION

$${}^n P_r = \frac{n!}{(n-r)!}.$$

$${}^n C_r = \frac{n!}{r!(n-r)!}.$$

$$n! = 1.2.3.\dots.n.$$

$${}^n C_r = {}^n C_{n-r}.$$

$${}^n C_r + {}^n C_{r-1} = (n+1) C_r.$$

$$(m+n)C_r = \frac{(m+n)!}{m!n!}.$$

BINOMIAL THEOREM

$$(x+a)^n = x^n + {}^n C_1 x^{n-1} a + {}^n C_2 x^{n-2} a^2 + {}^n C_3 x^{n-3} a^3 + \dots + {}^n C_n a^n.$$

$$n^{\text{th}} \text{ term, } T_{r+1} = {}^n C_r x^{n-r} a^r.$$

PARTIAL FRACTIONS

$\frac{f(x)}{g(x)}$ is a proper fraction if the $\text{deg}(g(x)) > \text{deg}(f(x))$.

$\frac{f(x)}{g(x)}$ is an improper fraction if the $\text{deg}(g(x)) \leq \text{deg}(f(x))$.

1. Linear non-repeated factors

$$\frac{f(x)}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{(cx+d)}.$$

2. Linear repeated factors

$$\frac{f(x)}{(ax+b)(cx+d)^2} = \frac{A}{ax+b} + \frac{B}{(cx+d)} + \frac{C}{(cx+d)^2}.$$

3. Non-linear (quadratic which can not be factorized)

$$\frac{f(x)}{(ax^2+b)(cx^2+d)} = \frac{Ax+B}{ax^2+b} + \frac{Cx+D}{(cx^2+d)}.$$

ANALYTICAL GEOMETRY

1. Distance between the two points (x_1, y_1) and (x_2, y_2) in the plane is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{OR} \quad \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

2. Section formula

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right) \quad (\text{for internal division}),$$

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n} \right) \quad (\text{for external division}).$$

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3. Mid point formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

4. Centroid formula

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right).$$

5. Area of triangle when their vertices are given,

$$\begin{aligned} & \frac{1}{2} \sum x_1(y_2 - y_3) \\ &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \end{aligned}$$

STRAIGHT LINE

Slope (or Gradient) of a line = tangent of an inclination = $\tan\theta$.

Slope of a X- axis = 0

Slope of a line parallel to X-axis = 0

Slope of a Y- axis = ∞

Slope of a line parallel to Y-axis = ∞

Slope of a line joining (x_1, x_2) and $(y_1, y_2) = \frac{y_2 - y_1}{x_2 - x_1}$.

If two lines are parallel, then their slopes are equal ($m_1 = m_2$)

If two lines are perpendicular, then their product of slopes is -1 ($m_1 m_2 = -1$)

EQUATIONS OF STRAIGHT LINE

1. $y = mx + c$ (slope-intercept form)

$y - y_1 = m(x - x_1)$ (point-slope form)

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ (two point form)

$\frac{x}{a} + \frac{y}{b} = 1$ (intercept form)

$x \cos\alpha + y \sin\alpha = P$ (normal form)

Equation of a straight line in the general form is $ax^2 + bx + c = 0$

Slope of $ax^2 + bx + c = 0$ is $-\left(\frac{a}{b}\right)$

2. Angle between two straight lines is given by, $\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

Length of the perpendicular from a point (x_1, x_2) and the straight line $ax^2 + bx + c$

$= 0$ is $\left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

Equation of a straight line passing through intersection of two lines $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ is $a_1x^2 + b_1x + c_1 + K(a_2x^2 + b_2x + c_2) = 0$, where K is any constant.

Two lines meeting a point are called intersecting lines.

More than two lines meeting a point are called concurrent lines.

Equation of bisector of angle between the lines $a_1x + b_1y + c_1 = 0$ and

$$a_2x + b_2y + c_2 = 0 \text{ is } \frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

PAIR OF STRAIGHT LINES

1. An equation $ax^2 + 2hxy + by^2 = 0$, represents a pair of lines passing through origin generally called as homogeneous equation of degree 2 in x and y and

$$\text{angle between these is given by } \tan\theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|.$$

$ax^2 + 2hxy + by^2 = 0$, represents a pair of coincident lines, if $h^2 = ab$ and the same represents a pair of perpendicular lines, if $a + b = 0$.

If m_1 and m_2 are the slopes of the lines $ax^2 + 2hxy + by^2 = 0$, then $m_1 + m_2 = -\frac{2h}{b}$

$$\text{and } m_1 m_2 = \frac{a}{b}.$$

2. An equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is called second general second order equation represents a pair of lines if it satisfies the condition

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0.$$

The angle between the lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is given by

$$\tan\theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|.$$

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, represents a pair of parallel lines, if $h^2 = ab$ and $af^2 = bg^2$ and the distance between the parallel lines is

$$\left| \frac{2\sqrt{g^2 - ac}}{a(a + b)} \right|.$$

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, represents a pair of perpendicular lines, if $a + b = 0$.

TRIGNOMETRY

Area of a sector of a circle = $\frac{1}{2}r^2\theta$.

Arc length, $S = r\theta$.

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, $\tan\theta = \frac{\text{opp}}{\text{adj}}$, $\cot\theta = \frac{\text{adj}}{\text{opp}}$, $\sec\theta = \frac{\text{hyp}}{\text{adj}}$, $\text{cosec}\theta = \frac{\text{hyp}}{\text{opp}}$.

$\text{Sin}\theta = \frac{1}{\text{cosec}\theta}$ or $\text{cosec}\theta = \frac{1}{\sin\theta}$, $\cos\theta = \frac{1}{\sec\theta}$ or $\sec\theta = \frac{1}{\cos\theta}$,

$\tan\theta = \frac{1}{\cot\theta}$ or $\cot\theta = \frac{1}{\tan\theta}$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$.

$\sin^2\theta + \cos^2\theta = 1$; $\Rightarrow \sin^2\theta = 1 - \cos^2\theta$; $\cos^2\theta = 1 - \sin^2\theta$;

$\sec^2\theta - \tan^2\theta = 1$; $\Rightarrow \sec^2\theta = 1 + \tan^2\theta$; $\tan^2\theta = \sec^2\theta - 1$;

$\text{cosec}^2\theta - \cot^2\theta = 1$; $\Rightarrow \text{cosec}^2\theta = 1 + \cot^2\theta$; $\cot^2\theta = \text{cosec}^2\theta - 1$.

STANDARD ANGLES

	0^0 or 0	30^0 or $\frac{\pi}{6}$	45^0 or $\frac{\pi}{4}$	60^0 or $\frac{\pi}{3}$	90^0 or $\frac{\pi}{2}$	15^0 or $\frac{\pi}{12}$	75^0 or $\frac{5\pi}{12}$
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$\frac{\sqrt{3}-1}{\sqrt{3}+1}$	$\frac{\sqrt{3}+1}{\sqrt{3}-1}$
Cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$\frac{\sqrt{3}+1}{\sqrt{3}-1}$	$\frac{\sqrt{3}-1}{\sqrt{3}+1}$
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	1	∞	$\frac{2\sqrt{2}}{\sqrt{3}+1}$	$\frac{2\sqrt{2}}{\sqrt{3}-1}$
Cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\frac{2\sqrt{2}}{\sqrt{3}-1}$	$\frac{2\sqrt{2}}{\sqrt{3}+1}$

ALLIED ANGLES

Trigonometric functions of angles which are in the 2nd, 3rd and 4th quadrants can be obtained as follows :

If the transformation begins at 90^0 or 270^0 , the trigonometric functions changes as

$\sin \leftrightarrow \cos$

$\tan \leftrightarrow \cot$

$\sec \leftrightarrow \text{cosec}$

where as the transformation begins at 180^0 or 360^0 , the same trigonometric functions will be retained, however the signs (+ or -) of the functions decides ASTC rule.

COMPOUND ANGLES

$$\sin(A+B) = \sin A \cos B + \cos A \sin B.$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B.$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B.$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B.$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\tan\left(\frac{\pi}{4} + A\right) = \frac{1 + \tan A}{1 - \tan A}$$

$$\tan\left(\frac{\pi}{4} - A\right) = \frac{1 - \tan A}{1 + \tan A}$$

$$\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - (\tan A \tan B + \tan B \tan C + \tan C \tan A)}$$

$$\sin(A+B) \sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$$

$$\cos(A+B) \cos(A-B) = \cos^2 A - \sin^2 B$$

MULTIPLE ANGLES

$$1. \sin 2A = 2 \sin A \cos A. \quad 2. \sin 2A = \frac{2 \tan A}{1 + \tan^2 A}.$$

$$\begin{aligned} 3. \cos 2A &= \cos^2 A - \sin^2 A \\ &= 1 - 2 \sin^2 A. \\ &= 2 \cos^2 A - 1 \\ &= \frac{1 - \tan^2 A}{1 + \tan^2 A} \end{aligned}$$

$$4. \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}, \quad 5. 1 + \cos 2A = 2 \cos^2 A, \quad 6. \cos^2 A = \frac{1}{2}(1 + \cos 2A).$$

$$7. 1 - \cos 2A = 2 \sin^2 A, \quad 8. \sin^2 A = \frac{1}{2}(1 - \cos 2A), \quad 9. 1 + \sin 2A = (\sin A + \cos A)^2,$$

$$10. 1 - \sin 2A = (\cos A - \sin A)^2 = (\sin A - \cos A)^2, \quad 11. \cos 3A = 4 \cos^3 A - 3 \cos A,$$

$$12. \sin 3A = 3 \sin A - 4 \sin^3 A, \quad 13. \tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}.$$

HALF ANGLE FORMULAE

$$1) \sin \theta = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}. \quad 2) \sin \theta = \frac{2 \tan \left(\frac{\theta}{2} \right)}{1 + \tan^2 \left(\frac{\theta}{2} \right)}. \quad 3) \cos \theta = \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}.$$

$$4) \cos \theta = 1 - 2 \sin^2 \frac{\theta}{2}. \quad 5) \cos \theta = 2 \cos^2 \frac{\theta}{2} - 1. \quad 6) \cos \theta = \frac{1 - \tan^2 \left(\frac{\theta}{2} \right)}{1 + \tan^2 \left(\frac{\theta}{2} \right)}.$$

$$7) \tan \theta = \frac{2 \tan \left(\frac{\theta}{2} \right)}{1 - \tan^2 \left(\frac{\theta}{2} \right)}. \quad 8) 1 + \cos \theta = 2 \cos^2 \frac{\theta}{2}. \quad 9) 1 - \cos \theta = 2 \sin^2 \frac{\theta}{2}.$$

PRODUCT TO SUM

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B).$$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B).$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B).$$

$$2 \sin A \sin B = \cos(A+B) - \cos(A-B).$$

SUM TO PRODUCT

$$\sin C + \sin D = 2 \sin \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right).$$

$$\sin C - \sin D = 2 \cos \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right).$$

$$\cos C + \cos D = 2 \cos \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right).$$

$$\cos C - \cos D = -2 \sin \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right).$$

OR

$$\cos C - \cos D = 2 \sin \left(\frac{D+C}{2} \right) \sin \left(\frac{D-C}{2} \right).$$

PROPERTIES AND SOLUTIONS OF TRIANGLE

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$, where R is the circum radius of the triangle.

Cosine Rule: $a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$,

$$b^2 = a^2 + c^2 - 2ac \cos B \text{ or } \cos B = \frac{a^2 + c^2 - b^2}{2ac},$$

$$c^2 = a^2 + b^2 - 2ab \cos C \text{ or } \cos C = \frac{a^2 + b^2 - c^2}{2ab}.$$

Projection Rule: $a = b \cos C + c \cos B$
 $b = c \cos A + a \cos C$
 $c = a \cos B + b \cos A$

Tangents Rule:

$$\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c} \cot\left(\frac{A}{2}\right),$$

$$\tan\left(\frac{C-A}{2}\right) = \frac{c-a}{c+a} \cot\left(\frac{B}{2}\right),$$

$$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \cot\left(\frac{C}{2}\right).$$

Half angle formula:

$$\sin\left(\frac{A}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{bc}}, \quad \cos\left(\frac{A}{2}\right) = \sqrt{\frac{s(s-a)}{bc}}, \quad \tan\left(\frac{A}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

$$\sin\left(\frac{B}{2}\right) = \sqrt{\frac{(s-a)(s-c)}{ac}}, \quad \cos\left(\frac{B}{2}\right) = \sqrt{\frac{s(s-b)}{ac}}, \quad \tan\left(\frac{B}{2}\right) = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}.$$

$$\sin\left(\frac{C}{2}\right) = \sqrt{\frac{(s-a)(s-b)}{ab}}, \quad \cos\left(\frac{C}{2}\right) = \sqrt{\frac{s(s-c)}{ab}}, \quad \tan\left(\frac{C}{2}\right) = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}.$$

$$\text{Area of triangle ABC} = \sqrt{s(s-a)(s-b)(s-c)},$$

$$\text{Area of triangle ABC} = \frac{1}{2} bc \sin A = \frac{1}{2} ac \sin B = \frac{1}{2} ab \sin C.$$

LIMITS

1. If $f(-x) = f(x)$, then $f(x)$ is called **Even Function**
2. If $f(-x) = -f(x)$, then $f(x)$ is called **Odd Function**
3. If P is the smallest +ve real number such that if $f(x+P) = f(x)$, then $f(x)$ is called a **periodic function** with period P .
4. Right Hand Limit (RHL) = $\lim_{x \rightarrow a^+} (f(x)) = \lim_{h \rightarrow 0} (f(a+h))$

$$\text{Left Hand Limit (LHL)} = \lim_{x \rightarrow a^-} (f(x)) = \lim_{h \rightarrow 0} (f(a-h))$$

If RHL=LHL then $\lim_{x \rightarrow a} (f(x))$ exists and

$$\lim_{x \rightarrow a} (f(x)) = \text{RHL} = \text{LHL}$$

5. $\lim_{n \rightarrow \infty} \frac{1}{n^p} = 0$, if $p > 0$ and $\lim_{n \rightarrow \infty} n^p = \infty$ if $p > 0$
6. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{\tan x}{x}$ (x in radians) $= \lim_{x \rightarrow 0} \frac{x}{\sin x} = \lim_{x \rightarrow 0} \frac{x}{\tan x} = 1$
7. $\lim_{x \rightarrow 0} \frac{\sin x^0}{x} = \lim_{x \rightarrow 0} \frac{\tan x^0}{x} = \frac{\pi}{180}$
8. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x}{x} = \frac{2}{\pi}$
9. $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = 1 = \lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x}$
10. $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$, where n is an **integer** or a **fraction**.
11. $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a$, $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \log e = 1$
12. $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$, $\lim_{x \rightarrow 0} (1 + n)^{\frac{1}{n}} = e$
13. $\lim_{x \rightarrow a} [kf(x)] = k \lim_{x \rightarrow a} f(x)$
14. $\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$
15. $\lim_{x \rightarrow a} f(x) \cdot g(x) = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} \left(\frac{f(x)}{g(x)} \right) = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$ provided $\lim_{x \rightarrow a} g(x) \neq 0$
16. A function $f(x)$ is said to be **continuous** at the point $x = a$ if
- (i) $\lim_{x \rightarrow a} f(x)$ exists (ii) $f(a)$ is defined (iii) $\lim_{x \rightarrow a} f(x) = f(a)$
17. A function $f(x)$ is said to be **discontinuous or not continuous** at $x = a$ if
- (i) $f(x)$ is not defined at $x = a$ (ii) $\lim_{x \rightarrow a} f(x)$ does not exist at $x = a$
- (iii) $\lim_{x \rightarrow a+0} f(x) \neq \lim_{x \rightarrow a-0} f(x) \neq f(a)$
18. If two functions $f(x)$ and $g(x)$ are continuous then $f(x) + g(x)$ is continuous